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The Contribution of Mathematics to General Education*

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WHILE I shall speak on the contribution to general education of mathematics, I have no intention of defending or glorifying all of the practices of past or present mathematical instruction. I myself have criticized sharply some of the traditional methods and some of the traditional subject-matter of mathematical courses in all grades of mathematical instruction, and I have pointed out the fallacies which lurk in claims for general improvement in logical power or in ability to think which have been made by some enthusiastic teachers of mathematics. More particularly, I have insisted that the teaching of mathematics for so-called mental discipline is not supported by sound psychological investigation. Indeed, I have pointed out that transfer of training even within the boundaries of mathematics itself is limited in its scope.

Nevertheless, I do think, and I believe that the majority of fair-minded educational leaders agree, that mathematical training does have a distinct place in modern education, particularly if the teaching of it, and the selection of the subject matter taught, conforms to the best principles of education, that is, to the real needs of the individual and of society.

Such conclusions are reached, as I shall point out, by the Educational Policies Commission of the National Education Association, whose Report on the Purposes of Education in American Democracy, published last year, is representative of the most liberal advanced thought on education. While I shall indicate substantial approval of, and agreement with, that Report, I shall feel free to depart from it in details. In general, however, I shall base what I have to say largely upon that Report.

The Commission has emphasized the contribution which education can and should make to the maintenance of democracy and of democratic ideals, which are threatened now as never before throughout the world. In this we must all agree, and I myself subscribe emphatically to this doctrine. The nature of democracy itself, its relation to society, to the individual, and to the school, is admirably presented in this Report, which is doubtless familiar to all of you.

One danger that may exist, and which seems to have been operative in some dis-

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cussions, is that the excellent presentation made by the Commission, and the present wave of strong feeling against dictatorial governments, may lead to an overemphasis on democracy and on social conditions which I do not believe was intended by the Commission. A careful reading of the Report will show that the Commission does not intend that these topics, important as they are, shall constitute the entire school program, or that other subjects shall be eliminated from all schools.

Thus the Theses on "The School and the State in American Democracy" presented at the National Council of Education last February by Professor Counts appears to me to exceed considerably the intention of the Commission. Professor Counts seems to propose that all teachers unite in a crusade quite independent of the rest of the citizenry, in support of a democratic ideal which he himself defines, and with detailed programs which can hardly be accepted even by all liberally minded persons. I can not agree with such an exaggerated program, nor do I think that it represents correctly the intention of the Commission in its Report. We should all agree to defend and to promote democracy as it is customarily defined, and to give every attention to every aspect of social welfare which meets with general approval. This will not mean, however, exclusive attention to social problems, as will be evident from even casual reading of the Report, nor will it entail acceptance by all teachers of the social views of even so noted an educator as Professor Counts.

Overemphasis upon even such a vital matter as the maintenance of democratic institutions will defeat its own purpose. Surely any demand that all teachers conform to a prescribed set of beliefs is utterly beyond the intention of any friends of true democracy. Teachers themselves must have and must maintain in their own schools their individual rights to freedom of thought and freedom of speech. Any other practice would be indicative of loss of democracy in the schools themselves.

The objectives of education outlined by the Commission emphasize the importance of the individual and his needs, and stress the view that democracy places the individual and his rights at the head of all rights. Thus on page 17, there is a main heading which reads "The General Welfare Places Individuals above Institutions." While we are striving to teach democracy and its preservation for the good of society at large, we must not neglect, nor does this Report neglect, the individual as the center of our attention. The individual can not take a selfish attitude toward Society without inviting disaster, nor can Society take a selfish attitude toward the individual in a real democracy. If individuals pursue only selfish ends, they may soon find themselves in a Society in which life is not worth living. If Society neglects the individual, democracy is automatically extinguished. In what I shall say today, therefore, I shall try not to neglect the rights and interests of Society, on the one hand, nor of the individual on the other hand. I wish therefore to treat the contribution of mathematics to general education under these two heads: the contribution to the individual, and the contribution to Society.

The recognition of the usefulness of elementary mathematics to the individual in the Report of the Commission is unequivocal. I can do no better than to quote directly from the Report. On page 58 of the Report, the Commission says:

"Some acquaintance with numbers and skill in the fundamentals of addition, subtraction, multiplication, and division is an educational objective to be taken for granted. The skills to be taught in this field and the types of problems to which these skills are applied should be determined by the kinds of arithmetical calculations which the ordinary American citizen has occasion to make. Elaborate and helpful investigations have been made to bring these fundamental operations into a position of prominence and recent

revision of the curriculum in many school systems has resulted in great improvement in Arithmetic instruction. In addition to skill in mathematics, there needs to be developed an appreciation of the cultural value of mathematics, and of its usefulness as a mode of thinking and as a means of interpreting world affairs.

"Closely associated with the fundamental arithmetical operations are the elements of intuitional geometry and applied algebra. Intensive technical study of more advanced mathematics should be offered to those whose vocational outlook, future education, or other special interests will make it necessary or helpful for them to use such knowledge.

"New aspects of applied mathematics are constantly developing and the educational experiences of children and adults need to be extended to include them. For example, the presentation of numbers in graphic and tabular form is becoming extremely common. Children should learn the rudiments of graphic presentation, particularly since this form of presentation is at once so effective and so easy to misinterpret. The presentation of numerical data in graphic form is becoming a language with its own grammar and syntax. It is, however, a language which can ensnare and deceive the unwary. . . .

"The ability to deal with number and form, the fundamentals of mathematics, has always been a basic human need. In an age such as ours where almost every phase of life is strongly marked by applied science and technology, the appreciation and use of basic mathematical skills and concepts offer significant assistance for self-realization."

As an exponent of mathematics, I see no need to quarrel with that excellent statement. If the schools carry out such a program in good faith, without prejudice and without omission of anything implied in the statement, no friend of mathematics who is informed on psychology and education will have anything but praise to offer. The implications of the program are more far-reaching than some may suppose, however, and it will not constitute unprejudiced adherence to the Report to include only the minimum explicitly stated without including the far-reaching implications. An extended Report to which the Commission here refers is another report entitled "The Place of Mathematics in Secondary Education" by a committee of which Professor K. P. Williams was the Chairman. A conscientious study of the quotation which I have made from the Educational Policies Commission and of the other report just mentioned will form a sufficient basis for sound work in elementary and in secondary schools; but I emphasize that the study and the application of the study to schools must indeed be conscientious if it is to carry out without prejudice the real intention.

I can not take the time here to elaborate the implications of this program, nor shall I attempt to add greatly to it, except by way of illustration.

I shall illustrate the danger that omissions may be made in good faith which ought not to be made by a quotation from the Report itself. On page 147, after a brief mention of serious difficulties that confront us all, the Commission says:

"And what are the children in this school, in this age, in this culture, learning? They are learning that the square of the sum of two numbers equals the sum of their squares plus twice their product; that Millard Fillmore was the thirteenth President of the United States and held office from January 10, 1850 to March 4, 1853, that the capital of Honduras is Tegucigalpa, that there were two Peloponnesian Wars and three Punic Wars, and that Latin verbs meaning command, obey, etc., take the dative." . . . "For the great majority of the boys and girls who now attend American schools such learning is transitory and of extremely little value."

It is evident that the Commission did not intend here to condemn all of the several topics from which these items are chosen: mathematics, U. S. History, geog-

raphy, ancient History, language. Evidently the intention was to select some very bad item from each one: thus the capital of Honduras was mentioned instead of the capital of France or of Germany. The item selected from mathematics should then have been a particularly bad item. I wish to use it to show two things. First, perhaps, that transfer of training is quite difficult even within the bounds of a single subject. Second, that caution must be used in the elimination of a given item without careful examination. There is no doubt whatever, from the context, that the person who selected the mathematical item did not transfer his own knowledge of it to mathematical things of the common world.

I may begin by relating an experience at the beginning of the World War. I was then acting as Chairman of the Committee on Rooms for my University, and I was suddenly confronted by an Army Sergeant who had been sent to measure the cubical contents of rooms for the army, under regulations concerning the number of cubic feet of air-space required per soldier. I saw that he was not measuring at all, but estimating by eye. When I demurred, he told me that he could judge such distances to within one foot in each dimension, so that his error in the cubic feet capacity would not be more than one cubic foot.—Is his mistake obvious to you?—Is the connection with the previous mathematical item obvious?—If not, then you show difficulty of transfer of training.

I have, indeed, used the *cube* of a sum in place of the *square* of a sum, and I have thought of the room as essentially cubical. The estimate made by the sergeant on a room 30 ft. cube would have been approximately 2700 cu. ft. instead of *one* cu. ft., as you will see by easy calculation.

People are continually making the same error. The simplest instance is that an error made in measuring the side of a square is more important than most persons imagine. A variety of such cases arise by ignorance or by design. If an ordinary rug which is supposed here to be about 12 ft. square (for simplicity) is six inches over or under size in each direction, the despised formula $(a+b)^2=a^2+2ab+\cdots$ shows that the *area* is really *twelve* sq. ft. over or under size, very nearly. Now the *area* of a rug of a given quality is what the householder is paying for. Articles are frequently made slightly under size to deceive the customer who does not know how to think this out.

May I dwell a moment on another instance, apparently far removed from this one? In multiplying two numbers together, people frequently carry out the multiplication to more places than they should. When measurements of any sort are taken, the assumption is that the last figure stated is correct to within five units in the next decimal place. For simplicity again, suppose that the numbers are equal, and that each one is 84. The presumed error is then 0.5; an application of the despised formula shows that the error in the product may be as large as 84, since $2ab$ is $2(0.5)(84)$. To write the answer as 7056 is therefore improperly accurate.

Uses of squares and cubes abound in ordinary life. Thus I have often called attention to the fact that cans containing food in grocery stores are usually made taller than they are wide in order to give a deceptive appearance of contents. The ordinary housewife, seeing two cans of the same width, one of which is twice as tall as another, concludes that the taller contains twice as much; and she is right. If she sees two cans, of the same height, one of which is twice as wide as the other, she will conclude that the wider one contains twice as much; and again she is right! In fact, the wider one contains *four* times as much. This results quickly from that despised formula. I may say that every tall can in every grocery store is a monument to the women who "*do not need to know any algebra.*"

Volumes of like solids are proportional to the cubes of their dimensions. A curious instance that still baffles me is that all o

the recipes that I have seen in cook-books call for a given *number* of eggs, for example, without regard to size. Women buy eggs strictly by number. Suppose that of two eggs of similar shapes, one is one-fifth shorter than another. That is, the lengths are as four is to five. Such eggs exist. Had you ever stopped to realize that the contents of such eggs are in the ratio of sixty-four to one hundred twenty-five? That is, the one egg contains very nearly twice as much as the other. How in the world can the cook-books be even moderately accurate? Why in the world do women accept twelve eggs of the one size at the same price as twelve of the other size? Is it because somebody told them that the formula for $(a+b)^2$ and for $(a+b)^3$ are not of any considerable value to them? If you do not commonly buy eggs, perhaps you do buy oranges. Maybe the oranges have thick skins. If they do, the formulas for $(a+b)^3$ may warn you in time; unless, of course, you don't care what you get for your money. Selling oranges that are slightly under-sized to people who have been told not to bother about $(a+b)^3$ is profitable.

I have dwelt upon this illustration because it does bring out the two points which I mentioned. Transfer of training is difficult: I know a considerable number of teachers of algebra who do not recognize such instances of the square (or the cube) of a sum unless it is specifically pointed out to them. Surely we must tell students such instances when we teach such formulas, or the formula is, in fact, "of extremely little value." On the other hand, the very fact that there is this difficulty of transfer of training makes it necessary to exercise great caution in stating that a given item is not of practical value; thus I believe that the person who selected this item $(a+b)^2$ as an example of a particularly useless item in algebra was not aware of its applications because his own knowledge of the formula did not transfer to practical situations.

My conclusions are two-fold. In the first place, I have long believed and maintained that teachers of mathematics must search out and must present to students a variety of situations in which a given mathematical item is applicable. If this is not done, the difficulties of transfer of training will prevent the student from making any great use of the item. To my own mind, the failure of teachers of mathematics to do this is one of the major shortcomings in the whole field of mathematical teaching. In the second place, however, educators and builders of curricula have often taken at face value the worst possible presentations of mathematical material, and they have frequently adjudged an item to be useless which is, in fact, of very frequent application to the affairs of daily life. Perhaps we who are teachers of mathematics are responsible for this, but I do think that educators and builders of curricula should look at items with greater care before they reject them. It is not enough to say that no uses of a given item were shown to you by your teacher, nor is it enough to say that useful applications do not occur to you readily without effort. It should be borne in mind, particularly by educators, that transfer of training is not easy, it is difficult; such applications are not commonly thought about unless somebody—either the teacher or the educational expert—really exerts himself.

The instance that I have used is not unique. I have made it my business to try to discover what uses there are for given items in mathematics, and what failures to transfer do exist in the minds of teachers and of students. I can take your time now only to assure you that such a search is very revealing, and to urge that every teacher of mathematics (and, too, every critic of mathematics) should bestir himself on every item before he disposes of that item. In another connection, I shall mention briefly some other instances, by way of illustration.

The special needs of experts, and the needs of those who intend to proceed to technical study, must not detain me here.

The statement made by the Commission is sufficiently broad to cover these necessities fully, if school authorities will follow those recommendations without prejudice and without misconstruction. I shall limit myself to general education, as my topic implies.

The other side of general education which deserves serious attention is the need of society as a whole for an educated citizenry. May I turn your attention for a few moments to those questions of public concern which require mathematical knowledge for intelligent decision.

Among such public questions, I shall emphasize those of *insurance*, of *installment buying*, of *bond issues*, and of *pensions*. I believe that all will admit that these are serious questions of public policy at this time. While I shall speak of them from this social standpoint, it will be evident also that many individuals are tremendously affected by each of these questions, so that I might have discussed several of them under the head of individual needs, rather than under my present heading of public needs.

It is very unfortunate that a large part of the public feels hopelessly ignorant of the methods of computation of any one of these things, in spite of the tremendous private and public impact of them on our lives and on our society. It is interesting, and curiously true, that the key to each of these questions lies in compound interest.

I shall not burden you with long arguments or with the derivations of formulas. Those of you who know them already will recognize them instantly without proof, and those to whom they may be new would not be edified by a very quick proof. It is sufficient to say that the total amount produced at compound interest from any principal p after n years is

$$p(1+r)^n,$$

where r is the rate of interest per annum. Here appears again the power of a binomial, whose uses I have dwelt upon above in other instances. To appreciate at all the problems that arise in insurance, in installment buying, in bond issues, in pensions, citizens must realize something of the manner in which this formula works. If they do not, they can not form any intelligent judgements whatever. Not that every citizen should know how to calculate in detail each of these things. But simple instances and—so to speak—sample calculations of compound interest give the only possible clue to the workings of these things.

Moreover, if payments are made each year, as in installment purchases of property, or in pensions, the aggregated amounts form a sum of terms like that above, that is, a sum of powers of $(1+r)$ raised to successive exponential powers. This constitutes at once what is known to every person who has studied algebra as a *geometric progression*.

I have no desire to proceed further with any calculation. What I do desire is to point out that the two fundamental ideas, namely that of compound interest and that of geometric progression, are studied in all courses in algebra, and that these are the tools required for any comprehension of the four vital public questions which I have mentioned. No other serious difficulties exist in elementary comprehension of these great questions.

My desire is simply to point out that the two criticisms which I made in a different connection above, exist here also. One criticism is that teachers of mathematics have not sufficiently emphasized these things to students, and students do not transfer their training to enable them to comprehend these things unless teachers do point out these applications and carry out simple cases as illustrations. The other criticism is that educators and builders of curricula have themselves not transferred their own knowledge of these mathematical items sufficiently to realize that they do form the whole basis for public comprehension of these important public issues. Both the teacher and the educator should

realize that transfer of such mathematical items to such public questions is not easy. Both the teacher and the educator have a duty to seek out and to take into account such things.

Only a person of extremely great hardihood could assert that such public questions are not of vital importance to our society and to the individuals in that society. Only a person of very great ignorance could hold that even a mild comprehension of these public questions can be had without some widespread knowledge of the simpler phases of the mathematical items which I have mentioned. Only a person who is abysmally careless of the public good can hold that it is wise to have such public issues decided by voters who are wholly ignorant of the most elementary phases of them. Anyone who asserts that such mathematical items do not affect life and society is either ignorant or wilfully prejudiced, or both.

In the near future we face an election which will decide in California the issue of a very extensive old-age pension plan. We can approach such an issue wholly on the basis of our sympathies and our emotions, or we can form some mathematical judgement on at least an elementary basis. The future will bring before the voters many other public questions that involve just such issues. It is my strong hope that we shall have citizens who are informed, who do care, and who can at least lead the great mass of those not qualified to judge independently.

I have emphasized here specific instances of the influence of mathematics on general education, first from the standpoint of the individual and then from the standpoint of society. In each instance, I have tried to show that mathematical items which do apply in life may seem to be devoid of such application on account of the real difficulties of the transfer of training. I have emphasized the duty of the teacher of mathematics, in all these instances, to search out and to make clear in simple cases, the real applications of each such mathematical item. I have insisted also that the educator and the builder of curricula have a corresponding duty. Unless these leaders carry out their duties, the beneficial effects of mathematical training both for the individual and for society may be lost, in the one case because the teacher has not helped to make the transfer to the real situation, and in the other case because the educator has carelessly eliminated an item of importance because he himself had not made the transfer to realities.

What with my quotation from the Educational Policies Commission and the illustrative examples that I have given, I trust that I have presented an adequate framework which you yourselves may complete of the contribution that mathematics can make to general education.

THERE are obviously two educations. One teaches us how to make a living and the other how to live.—JAMES T. ADAMS.

MOST men are afflicted with a coldness, an incuriosity, as soon as any object does not connect with their self-love. Though they talk of the object before them, they are thinking of themselves, and their vanity is laying little traps for your admiration.—EMERSON.

Shall We Accept the Two Years of Required Academic Mathematics as a Right, a Ritual, or a Challenge?*

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TO PRESENT this question to a meeting of this kind seems rather like bearing coals to Newcastle. The mere fact that we have gathered here from all parts of the United States is in itself an answer to the question as far as you and I are concerned. That year after year these meetings are possible is because of our one interest in common, namely mathematics, and surely this is indicative of the fact that we have felt the challenge and are seeking an answer. Relatively speaking, however, we are a very small portion of that great body of teachers of mathematics. This brings us to the question, are we content to sit back and do nothing after having felt the challenge? To ask that question is more or less like the old darky camp meeting song with its repetition of "Has you been redeemed? Has you been redeemed?" and the every ready response, "Yas Lawd." We have come to this camp meeting either because we are seeking the light or because we have been redeemed or think we have. But have we? I wonder how many of us are active converts and how many are merely passive onlookers. The trouble is that just as the newly made convert either must take an active interest in his church or he is apt to fall from grace, so with us we either go forward or we fall back into old practices. That is one of the laws of human nature. I sometimes wonder how many of us, the lay members of our profession, are content to go through the exhilaration of being led to see the light only to drop back into the old rut once more. We forget the temporary thrill we received until the next camp meeting and then once more seek the mercy seat with the same lack of change. It becomes a rhythmic cycle of repentance and back sliding which is repeated over and over again. Does it not seem too bad that this pent up energy which was generated by the emotional drive of repentance was allowed to lapse unspent?

I think that each of us would answer the question presented by saying that he or she would accept this two years of required mathematics as a challenge but I wonder if there are not some of us who do so with a tongue in the cheek because of an inward certainty that while we accept it as a challenge, we will not allow any one to gainsay our right that these two units of mathematics should be required from the total offering of sixteen units which must be acquired by each pupil who would attain a high school diploma and entrance to the college of his choice. Then having assured ourselves that this is our right, there comes that inward feeling of security accompanied by a glowing sense of well being, permeated throughout with self justification so that once more it is safe to settle back into the same old ritual.

But can we truthfully say that we have a right to these two years of required mathematics? And if we can answer in the affirmative, then who can say how long this right will be ours? True, since Plato set up his college entrance requirements, with only temporary lapses, we have been so fortunate, or is it unfortunate, as to have some sort of mathematics as an entrance requirement of institutions of higher learning. At present a great majority of the colleges and universities

* A paper read before the National Council of Teachers of Mathematics and the Department of Secondary Education of the N.E.A. at San Francisco, July 4, 1938.

require one year each of algebra and geometry for entrance. However, many of us who are here have only to think back to our own academic training to remember that Greek and Latin were once required subjects also. And now where are they? Personally, I am glad that I was exposed to both of them but that does not alter the fact that Greek is practically unheard of in the secondary field and Latin has rapidly lost ground during the past two decades.

What has happened to Greek and Latin can happen to mathematics. This is no time for us to claim our vested rights, rather, as has been said, we must face facts. Can we prove that our subject does more properly prepare students for college than does any other? Messers Douglass, Gebhart, Stinnette, and others cite studies to convince us that those who lacked credits in mathematics were able to do just as satisfactory collegiate work as were those who had taken their required number of units in the subjects. On the other hand we find Mr. Hart and others who will show that the conclusions drawn by these former gentlemen are invalid. Probably you and I would be wiser to take the middle road and agree that right or wrong the baby is ours an we must do something about it or the child will die of a complication of malnutrition, paralysis or mayhap a touch of sheer boredom. In order to be agreeable, let us grant, for the moment, that we do have the right to these two years out of the life of every youngster who is knocking at the doors of our colleges. Then the street urchin in me ventures the question, "so what?" Are we to sit back supinely and "let the dead past bury its dead" or are we going to venture forth and keep abreast of the rapidly moving age in which we live?

Let us return to the original question. Shall we answer it with the second alternate, namely, ritual? It is true that no ritual of any of our most ancient fraternal organizations has been kept more sacredly pure from change than have many of our courses in secondary mathematics. This sanctity from intrusion by outside influence is no doubt a beautiful tribute to those great men of mathematics who gave so much to the development of our "Queen of the Sciences." However, I wonder how many of us would credit Euclid or Plato as having contributed more to the development of mathematics than we can credit to Thales or Archimedes. Had Thales been willing to accept the mathematics of the Egyptian priests as final, he never would have applied it in computing such things as a ship's distance from shore by means of congruent triangles nor used ratios in finding the heights of objects from their shadows. Had Archimedes been content with the ritual of Plato and Euclid and less interested in his dearly beloved curves, we might have waited many generations before we discovered the laws of hydro-statics, levers, et cetera. Now by these statements I do not mean to cast overboard as useless all of the contributions of Euclid and Plato. The former took the relatively unorganized geometric knowledge of his time and welded it into a logical chain that proceeded from certain assumptions which he treated as self evident. On the other hand the latter, it has been suggested, was useful if only because he stimulated greater mathematicians than himself to greater achievements than otherwise might have been theirs. There have been some heretics who have suggested that this stimulation may have been done chiefly through irritation, but we will ignore them and be thankful for the stimulation. Bell suggests that had the Greeks followed Archimedes instead of Euclid and Plato that they might easily have anticipated the age of modern mathematics which began with Descartes and Newton in the seventeenth century by at least two thousand years. No, a ritual will not do. Had there been no challenged assumptions, the progress of the past century would have been naught. When asked how he has been able to make his great discoveries, Einstein answered with a smile, "By challenging an assumption."

The only answer that I can wholeheartedly accept for this question is that the two years of required mathematics must be a challenge to us. We must give these pupils of ours something to take with them from their two years spent with us that will make them more able to cope with the problems which are before them than would otherwise have been possible. When I chose to take only the academic group for my theme, it was not because I did not feel the need for work to be done with the non-academic group. Indeed the field is there and it must be answered but my own peculiar interest has been with the forgotten pupils among the academic group. It has been taken for granted, until recently, that all pupils need two or more years of mathematics in the secondary school and so we have been more or less willing to rest our case after having made the usual plea of its value in mental discipline. Frankly, I cannot accept that theory. I do not feel that the findings of research studies will warrant our relying upon it. Transfer value will only come when we definitely teach for it and methinks all too few of us are doing that. We are trusting to some divine power to make this transfer for us. We expect transfer to take place without effort being put forth is too much like the city girl who went to the country to live. She was well grounded in the faith of her fathers' but had never learned the technique of turning on the tap of a cow's reservoir of milk. She, nevertheless, offered to do the milking and when they found her she was upon her knees diligently praying while the bucket was placed in the proper position to receive the milk. Now the cow was a kindly one with an udder full of milk. The girl had utmost faith in the value of milk as a food but she had never learned how to transfer the milk from the udder to the bucket. Now that milk was just as good milk in the udder as it was after it was transferred into the bucket but it would not have done as much good for the family. Now I contend that the pure mathematics that one stores in his brain is just as good mathematics as that which is applied but until the individual learns how to transfer it, it is not as valuable to mankind. I believe firmly in the power of faith but there comes to my mind another fact that "faith without works profiteth thee nothing." Just as among our forebears were some ingenious ones who learned to milk a cow through the use of common sense and cleverness so we will find some pupils who of their own volition and in spite of our teaching will recognize possibilities for transfer. I am inclined to feel, however, that the number is relatively small.

The fact that mathematics is required for entrance into most of the colleges in itself makes this not only a challenge but a real duty to perform. It challenges me to re-examine my own philosophy of general education. Then having done this I wish to study the aims of mathematics so that these aims will be in harmony with and fulfill the objectives set up in this general philosophy of education. If each of us will do this conscientiously, we need go no farther in our efforts to prove that mathematics has a very important gap to fill. And this gap by the way is fully as wide in the non-academic work as in the academic. I rather wonder if it is not because too many of our curriculum makers know so little about mathematics that we have had all of this talk about booting mathematics out of the curriculum.

To make a comprehensive survey of the philosophy of education would require far more time than is available at a meeting of this kind and would be worse than useless. For a philosophy to function in one's teaching, it is necessary that the individual develop one for himself through intensive individual research which is accompanied by hours of meditation. Probably each one of us has a well defined philosophy of education and no doubt these individual philosophies are surprisingly similar. Yet it would be a rather safe forecast that if we were to attempt to formulate a group philosophy here this afternoon, the argument would last far into the night. And it is most probable that

the differences would be trivial quibbles over phraseology. The Mathematics Committee of the Progressive Education Association has made a rather challenging statement of its philosophy of general education which will do as well as any other for us to use as a rack upon which we may hang either our bouquets or our brick bats. I use this for three reasons which are: first, while it is not stated exactly as I might, still I find that I can accept it as a starting point at least; second, I am a terribly sensitive soul and my feelings won't be hurt if you reject it, and third, I really think I have already given this panel enough vulnerable points at which they may fire at me personally.

The philosophy of the Mathematics Committee of the Progressive Education Association as stated in their Progress Report is in brief as follows: First, the schools must consider the basic aspects of living which may be divided into four overlapping and interdependent parts, namely, personal, immediate personal-social relationships, socio-civic relationships, and economic relationships. Second we must consider the educational values and related characteristics of personality which in turn may be subdivided into three parts which are, the recognition of the dignity and worth of the individual involving creativeness, appreciation, and wide range of interest, the effective social participation which involves social sensitivity and cooperativeness, and the use of that intelligence which will involve analyzing problem situations, readiness to act on the basis of tentative judgments and self-direction. It would be presumptious of me to attempt to tell you of the opportunity that this philosophy gives for the mathematics teacher.

When one re-examines his or her own philosophy in reference to these statements as made by the Progressive Education Association and at the same time considers the changing school population and changing college entrance requirements, it becomes necessary to ask one's self the same three questions which the mathematics committee asks concerning mathematics for the general student but applied to the academic pupil. First, is there useful mathematics available for those who are not science or engineering majors; second, what may be added that will not prove less interesting and useful to those who are expecting to need advanced mathematics for their life work; and third, is the mathematics being given such that will train these pupils to be useful citizens in this technological civilization in which we are now living? You can see that with this underlying philosophy, we will not find difficulty in filling the two years course. Rather we will be pressed for time.

You will note that in this paper no effort has been made to state what this one teacher has done to meet the challenge. That would be another speech. But you may rest assured that, coming as I do from a long line of pioneers and being thoroughly embued with a crusading spirit, I am trying to meet the challenge in my own classroom. To some I have seemed to be a rank heretic while others have classed me as a conservative. Although I think a thorough cleansing is needed in our departmental offerings, I agree with the educator who warns us that care must be taken lest we throw out the baby with the bath. So with at least one foot on the ground, may this one teacher remind you that to be truly progressive, one must be duly conservative.

QUESTIONS in regard to the program, presented by the Committee of the Private School Teachers Association and published in the November, 1939, issue of THE MATHEMATICS TEACHER, will be answered by the following members of the committee: Mrs. Irma Kaufman, The Bentley School, 48 West 86th Street, New York City, for the 7th and 8th grades; Miss Elizabeth Cooper, 157 East 72nd Street, New York City, for the 9th grade; Mrs. Cassius Keyser, Bamford-Nightingale School, 20–24 East 92nd Street, New York City, for the 10th grade; and Miss Helen Crossman, Packer Collegiate Institute, 170 Joralemon Street, Brooklyn, N. Y., for the 11th and 12th grades.

High School Mathematics in the University*

By H. M. Bacon
Stanford University, California

In the subject, "High School Mathematics in the Univeristy," I believe I have found a title which will permit me to discuss almost anything. For it might be interpreted to mean "what mathematics will the high school graduate need if he comes to the university," or it might be interpreted to mean "what courses in mathematics on the high school level should the university provide for the student who comes with an insufficient mathematics preparation"? Furthermore, although it may not be immediately obvious, I believe that this topic is in close accord with our general theme, "mathematics to meet social needs." Too often meeting social needs is thought to mean merely dashing about trying to find quick cures for deep-seated and little understood social ills, or making solemn pronouncements about the Social Order, or learning how to keep one's bank balance straight—to say nothing of learning how to acquire and keep a bank balance of any kind. But, if we are to make some attempt to solve the great social problems of our day, it is hardly open to question that we must have *knowledge*, not only about the so-called Social Sciences, but about everything which interests and influences people in their individual and social behavior. This means knowledge of just about everything under the sun. To get this knowledge, and to apply it intelligently to the solution of these problems requires serious and concentrated study. Much of it is most economically secured by studying at a college or university. To pursue such studies effectively, the student cannot dispense with at least some mathematics, and in some fields, he obviously cannot have too much mathematics. And there are even those who will study mathematics for its own sake. However, these are few in number; for, although mathematics is generally referred to as the *handmaiden of the sciences*, it is usually a case of "often a bridesmaid, but never a bride."

I shall not concern myself with those high school students who are not planning to enter the university: not that their problem is to be neglected, but that others more fully informed than I will discuss their situation. Let us think for a while of the young men and women who go on to college, ostensibly with the idea of rounding out their general education, preparing for a profession, or devoting their lives to teaching or to research in some chosen field.

Again, there is no need to speak further of those planning to study pure mathematics, engineering, the physical sciences, psychology, statistics, or other subjects which evidently require a full and thorough mathematical training in the high school. What may surpirse us is the plight of the boy who arrives at the university and wants to study economics or biology. He didn't like his mathematics at the high school, so he took the minimum amount. Or he found it difficult or exacting. Or he discovered that he had to do homework or some other series of unpleasant tasks which interfered with the integration of his personality at the neighborhood palace of the cinema. So he did not acquire even the most meager facility in his algebra, he didn't get any ideas about geometry except that his teacher insisted that he prove things which anybody could see were true anyhow. He goes to his economics class. The instructor draws a graph or two upon the board, he talks about the rate of change of a function,

* A paper read before the National Council of Teachers of Mathematics and the Department of Secondary Education of the N.E.A., July 4, 1939.

maybe he even writes an equation or two. What on earth is it all about anyway? It is clear that the instructor is terrible, and the text-book incomprehensible. In addition, the Scholarship Committee is adamant, and Johnny comes marching home to rusticate for a year. Perhaps the sequence of events is not quite so dramatic, but the inability to think in abstract terms, to analyse a problem, or even to grasp the difference between hypothesis and conclusion is too often the source of failure to benefit from the exposure to college work. Frequently, just the lack of acquaintance with the simplest algebraic technique adds hours of work to the load of any serious student of the social sciences. Just the other day, one of my student friends, a junior, came to ask me to show him how to solve four simultaneous linear equations. These had turned up in his accounting assignment. This is not mentioned as a criticism of his unfamiliarity with algebra (he had spent over two hours trying to work out a solution). It is to call attention to the fact that those students who may think they don't need mathematics in their economics and business courses are sadly mistaken.

The average young man is inclined to think of biology as fairly free of the taint of mathematics. But he won't get very far before he is confronted with growth curves and formulas, with statistical analyses, and algebraic calculations. If he brings some knowledge of, and some feeling for, these things from his preparatory school, he can be that much farther along the road to knowledge, and he has that much more time to devote to the advanced studies which he came to pursue. If only he listens to the advice of his high school mathematics teacher! But you and I know from experience that neither our most subtle hints nor our most vigorous counsel will reach *every* boy and girl.

Even the man who intends to eschew all sciences and wishes to delve only in the garden of the humanities cannot afford to avoid the fundamental mathematical training. Leaving aside the merely practical question of meeting the requirements which most colleges have set for the acquisition of the handsome diploma which assures the world that the recipient is an educated man, and which he may frame and hang upon the wall to foster and publicize the illusion, he must, if he is a serious student of the progress of mankind, pay some attention to mathematics —that most ancient and honorable field of human endeavor.

And if our young friend is coming to college merely to make *contacts*, to learn how to win friends and influence people, he still should know enough about arithmetic to keep the books of the multifarious organizations of which he is the treasurer, and enough about the theory of probability to restrain his tendency to put his father's money into a slot machine instead of the monthly board bill. He should know something about positive and negative numbers in order to escape the academic guillotine whose knife descends upon the neck of the unwary youth so careless as to be minus too many grade points, and he must, at the same time, know how to manage to pile up a sufficient number of units without wasting time acquiring any superfluous credits.

Thus, all joking aside, for all groups of college entrants, the better their mathematical background, the better equipped they are to accomplish their purposes, and the more advantageously can they employ their time. Our high school mathematics teachers realize this. They deserve every encouragement and every compliment for their efforts to have it understood and recognized.

Our problem therefore is: what is to be done for the student who arrives at the college with a minimum mathematical equipment—and this means virtually no equipment for the boy or girl who has been careless and indifferent to the advice and efforts of his mathematics teachers? Naturally enough, there is some divergence in theory and in practice in the answers

provided by various colleges and universities. Some give review courses in algebra and geometry with no credit allowed. Others require all students to take algebra and trigonometry regardless of high school background. A few require no mathematics whatever. It is impossible to survey all of the various plans, many of which are already familiar to you. Let me limit myself to describing the system with which I am best acquainted.

Quite a number of young men do not make up their minds that they want to enter engineering or some other field which requires professional training in mathematics until they actually reach the university. Most of these have taken only the customary two years of high school mathematics. It therefore seems incumbent upon the university to provide some means of making a quick review of elementary algebra, a course corresponding to the second year of algebra, and a course in trigonometry for these students. At Stanford an extra fee is charged for these courses. These were, until recently, the only mathematics courses available for the student who was not ready to proceed to the usual analytic geometry and calculus sequence, for which these courses or their high school equivalents form the preliminary work.

Now, it appears that a large number of the students who wish to study economics, biology, medicine (with consequent physics and chemistry), elementary accounting and statistics try to go ahead in these subjects with only the ordinary two year high school mathematics background. The result has been that courses in these subjects with any mathematical content—however simple—had to be scaled down to fit the preparation brought by such students. A gradual growth in demand for the establishment of some course in the Mathematics Department to provide a more adequate mathematical background for these people has been answered in the introduction of a brief course entitled "Elementary Mathematical Analysis." In this course we try, *in one quarter*, to give some notion of the meaning of *function*, of drawing and interpreting *graphs*, of finding the *rate of change of a function* and the *area under a curve*, first graphically, and then in the simplest cases by use of differential and integral calculus. Some exercise in algebraic manipulation comes in incidentally. The students who take this course generally do not intend to take any other mathematics, and they most certainly would not have taken the regular sequence of algebra, trigonometry, analytic geometry, and calculus covering from five to eight quarters.

It should be most particularly noted that the inspiration and urgent demand for this course came from those departments where a professional mathematics preparation is not required. We have had a large number of students come to this course on advice of members of the Law Department as well as of those already mentioned. Most of the students are eager and willing, and they work hard and faithfully. When they finish the one quarter's work, they have, of course, a very meager knowledge of and facility with mathematics, but it is better than nothing.

Imagine having one quarter in which to study all of the formal mathematics you are ever likely to see! If the enthusiasm and determination which most of these young people bring to this pitifully brief course were backed up by three or four years of high school mathematics under the guidance of such devoted teachers as are represented at this gathering, we could do great things for these non-professional mathematics students, even in one quarter! Let me come to the thought which I most want to leave with you.

For any high school boy or girl who is going on to the university, it is *more* and not *less* mathematics that is needed. In spite of excellent teaching, the two years ordinarily required in the high school is simply not enough, regardless of the eventual field of study chosen. It would be

difficult to put one's finger upon a more valuable asset to the college student than a real understanding of what constitutes the usual high school mathematics training—unless it is the ability to read thoughtfully and to write literate English. And let me add that it would be impossible for me to compliment too highly the splendid work which so many of our high school mathematics teachers are doing in preparing their students for the university. Further, let me say that all of us in universities appreciate the work which you are doing for the *cause of mathematics*, and therefore, we believe, for the cause of the true advancement of learning. May we continue to stand firmly together in this noble work and in our great calling.

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Number Concepts Held by Seven-Year-Olds

By AGNES G. GUNDERSON
University of Wyoming, Laramie, Wyoming

AFTER reading Whiteaker's article[1] in the January issue, one is challenged to make some attempt to discover what ideas or concepts children at different ages or grade levels have regarding number.

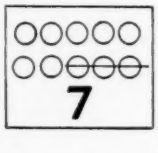
This study is a summary of an inventory of number ideas or concepts held by a group of seventeen children in our school who were completing the first half of Grade II. It was hoped this inventory might also give some indication of whether or not these children were ready for the more systematic development of addition and subtraction combinations which includes abstract number. At the time this inventory was taken no drill or formal arithmetic had been given. No combinations had been taught as such. All work had been informal—planned so as to enrich and extend children's experiences with number, and to make number meaningful to them.

The arithmetic period is usually twenty minutes a day. Our program, however, is a flexible one allowing for longer or shorter periods as the need demands. All work in arithmetic has been with concrete and semi-concrete materials up to this time. The following shows the type of arithmetic taught:

1. Counting by 1, 2, 5, 10.

2. Reading and writing numbers up to 100.

3. Grouping objects and comparing groups.



4. Playing store using real money. Each child is given one nickel, two nickels, or a dime to spend. He makes out his bill in picture form (draws 5 or 10 cents, crosses out the number of cents he spends, then writes the amount of change he should receive from the cashier). Later, when combinations are taught the bill will be made out using the combination form, e.g.

$$\begin{array}{r} 10¢ \\ -3¢ \\ \hline 7¢ \end{array}$$

5. A unit in time telling has been taught.

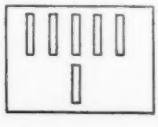
6. Roman numerals I–XII were taught in connection with time telling. Some children became interested in figuring out how numbers larger than XII were written. One child said, "I see them in chapters in books and I ask Mother what they are."

7. Only a little work in measuring has been given so far, such as using milk bottles to discover how much more a quart of milk is than a pint; using rulers to make calendars; measuring each other to determine their heights. This unit is to be continued.

8. Problem solving. Using objects such as cardboard squares, beans, buttons, the children have solved oral problems in addition and subtraction (sums and minuends not exceeding 10). They have also solved such problems using semi-concrete materials (dots, rings, lines).

9. The children are beginning to make their own problems in addition and subtraction, each problem accompanied by an illustration:

I had 5 books.
Mother gave me 1 more.
How many do I have now?



[1] Whiteaker, George H., "A Child's Concept of Numbers," THE MATHEMATICS TEACHER, 32: 25–26, January 1939.

In order to obtain as complete a picture as possible of the seven-year-old's concepts of the different numbers and fractional parts, the following introduction was given: We have been using numbers in playing store and other arithmetic work. It would be interesting to see how much you know about numbers. Let's take the number *two* first. Tell all you know about *two*. What is *two?* What does *two* mean? What does *two* make you think of? How can you show *two?* How can you get *two* from other numbers? Do you know any words that mean *two?* What things make you think of *two?*

The inventory was taken during the arithmetic period. The responses were given orally, except the Arabic and Roman numerals and fractional parts which were written on the blackboard.

This inventory reveals some interesting things: it shows that these seven-year-olds have had many contacts with the use and application of number in their daily work and play. They learn number concepts outside of the arithmetic class. The music class probably contributed the following terms: one-fourth note, solo, duet, etc. Some concepts are likely learned outside of school, as for instance, sizes and prices of clothing, measures used in buying groceries, such as milk, butter, etc.

The combinations taken up in Grade II are those whose sums or minuends are 10 or less, therefore, we were primarily interested in the concepts of the numbers 1 to 10 inclusive. After the numbers 1–12 had been taken up, one by one, the children were asked to tell about numbers larger than 12, these responses are given in Table III. Although this list is very incomplete it shows some interesting things, e.g. "1000 and 1000 are 2000." One wonders if the child thinks of "thousand" as if it were the name of the objects added. "Twelve 10's" was explained in terms of dimes. Many doubles of large numbers are known. The children have some knowledge of money. Only a few know the sizes of their coats, gloves, shirts and dresses, but almost all know the sizes of the shoes they wear, and also the distinction between child's and adult's sizes in shoes. Twelve know their weight but only five remember their height. They know their house and telephone numbers and read them as they are commonly read: (house no. 1317, thirteen, seventeen, phone 2156, two, one, five, six).

The group's understanding of fractional parts is shown in Table II. It is interesting to note that while their knowledge of one-third is very meager they seem to know one-half pretty well.

Abstract number had not been used in school up to this time, the teacher using concrete or semi-concrete materials, or names of objects, as 9 books, 4 boxes, 6 cents, etc.; yet the children used abstract number when given addition combinations, e.g. "4 and 3 are 7". When giving subtraction combinations they used the problem form as: "If you have 7 cents, and spend 5 cents, you will have 2 cents left." While in some instances they used the number in the abstract as 7 instead of 7 cents, they did not condense the problem to the mere statement—5 from 7 is 2, or 7 take away 5 is 2—as they did with the addition combinations. Is this an indication that they do not know subtraction as well as addition? Is the fact that they use abstract number in giving addition combinations evidence that they through many experiences with concrete and semi-concrete materials have reached the generalization that 3 and 2 are 5, etc.? The writer is inclined to think so.

It would seem from the responses given that this group of children understand the smaller numbers fairly well. Some, at least, seem ready for the simpler combinations. The work in arithmetic for the latter half of the year will continue to stress meanings and understandings, and the usefulness of number in daily life—which is Social Arithmetic.

In conclusion, let me say, this composite list or inventory is but an experiment. It is by no means exhaustive as the concepts

or items listed are only those which the child recalls. No doubt he would recognize many more than he is able to recall. Perhaps in time there will be available a list of minimum essentials in number concepts and arithmetical terms, similar to vocabulary tests, which the teacher might give as a readiness test before proceeding on to the next stage in arithmetic. Would not this be a worth-while contribution to the field of arithmetic teaching and a help to the teacher in grouping pupils?

TABLE I

NUMBER CONCEPTS

One

one
1 (Arabic numeral)
I (Roman numeral)
1 object, (apple, box, book, etc.)
first (ordinal)
First Grade
Grade I
counting by 1
1:00 o'clock (school begins)
1:30 o'clock (half hr. after 1:00)
1:15 o'clock (15 min. after 1:00)
1 month old (baby)
1 year old
1 is the beginning of number
only zero is less than 1
1 is 1 less than 2
1 is one-half of two
1 dime is 10¢
1 pie is a whole pie
1 pair (shoes, gloves, etc.)
solo (soloist in church)
3 is 2 more than 1
4 is 3 more than 1
8 is 7 more than 1
9 is 8 more than 1
12 is 11 more than 1
(The following subtraction facts given in problems)
$3-2=1$ $4-3=1$
If you have 2¢ and give away 1¢, you have 1¢ left

Two

two
2 (Arabic numeral)
II (Roman numeral)
2 objects
second (ordinal)
Second Grade
Grade II
counting to 2 (place in series)
counting by 2's
2:00 o'clock
2:30 o'clock
2 hands on clock
2 years old
2 floors in building
second floor
"two for a nickel"
pair (2 shoes in a pair)
couple
double
double decker bed
double decker bus
double wheels on truck
double door
double window
duet
twice
twins
twin beds
2 hands, feet, arms, etc.
2 halves in pie, apple, etc.
2 pencils in set
2 ends of a string
2 inches
2 pieces in cake
2 numbers in 21
20 has the number 2
2 wheels on bicycle, motorcycle
(Subtraction facts in problems)
$3-1=2$ $22-2=20$
(Addition facts—abstract number)
1 and 1 are 2

Three

three
3 (Arabic numeral)
III (Roman numeral)
3 objects
third (ordinal)
Third Grade
Grade III
counting to 3 (place in series)
3:00 o'clock (school is out)
3:30 o'clock
3 years old (too young for school)
3 letters in word (boy)
Third Street
third floor
three floors in building
three shelves in bookcase
3 panes in window
3 children in family
"The Three Bears"
"The Three Pigs"
3 numbers in 100
the number 33 has three's
3 ones in 111
three 1's are 3
thrice
trio
triplets
3 sides in triangle
3 wheels on airplane
3 wheels on tricycle
3 feet in a yard
3 dimes in 30¢
3 is more than 1, 2
(Subtraction facts in problems)
$5-2=3$ $4-1=3$
(Addition facts given, using abstract number)
2 and 1 are 3
1 and 2 are 3

Four

four
4 (Arabic numeral)
IV (Roman numeral)
4 objects
fourth (ordinal)
Fourth Grade
counting to 4 (place in series)
4:00 o'clock
4:30 o'clock (half hr. after 4)
4 years old
4 letters in word (ball)
Fourth Street
fourth floor
4 wheels on car, wagon, trailer
animals have 4 feet
4 corners in box (rectangle)
4 sides in square
4 weeks in a month
4 dimes in 40¢
2 pairs of shoes are 4 shoes
4 quarters in a dollar
fourths
quadruplets
quartet
children in room are about 4 ft. tall
4 numbers in 1000
the number 44 has two 4's
(Subtraction facts in problems)
$6-2=4$ $10-6=4$
$8-4=4$
(Addition facts) abstract number)
2 and 2 are 4
3 and 1 are 4
4 and 4 are 8

Five

five
5 (Arabic numeral)
V (Roman numeral)
5 objects
fifth (ordinal)
Fifth Grade
counting to 5
5:00 o'clock
5:30 o'clock
5 years old
5 letters in word (David)
counting by 5's
5 schooldays in week
5 fingers on one hand
5 toes on one foot

TABLE I—*Continued*

5 people in family
a nickel is 5¢
5 dimes in half a dollar
5 dimes are 50¢
the number 55 has two 5's
quintet
quintuplets
I had 3 pairs of shoes. I lost one shoe. I had 5 shoes left
(Subtraction facts in problems)
6 − 1 = 5 9 − 4 = 5
10 − 5 = 5 8 − 3 = 5
7 − 2 = 5
(Addition facts—abstract number)
3 and 2 are 5
2 and 3 are 5
1 and 4 are 5

Six

six
6 (Arabic numeral)
VI (Roman numeral)
6 objects
sixth (ordinal)
Sixth Grade
counting to 6
6:00 o'clock (supper)
6:30 o'clock
6 years old (start school)
3 is half of 6
half a dozen is 6
2 yards are 6 feet
6 dimes in 60¢
the number 666 has three 6's
6 is more than 1, 2, 3, 4, 5
(Subtraction facts in problems)
10 − 4 = 6 8 − 2 = 6
12 − 6 = 6
(Addition facts—abstract number)
3 and 3 are 6
4 and 2 are 6
5 and 1 are 6
2 and 4 are 6

Seven

seven
7 (Arabic numeral)
VII (Roman numeral)
7 objects
seventh (ordinal)
Seventh Grade
counting to 7 (place in series)
7:00 o'clock
7:30 o'clock (half hr. after 7:00)
7:15 o'clock (15 min. after 7:00)
7 years old (in Second Grade)
7 letters in word (morning)
8 years is one more than 7 years
7 comes after 6
7 is between 6 and 8
7 days in a week
January 7 (calendar)
1 pint of milk costs 7¢
the number 77 has two 7's
(Subtraction facts in problems)
9 − 2 = 7 11 − 4 = 7
10 − 3 = 7
Addition facts—abstract numbers)
4 and 3 are 7
6 and 1 are 7
5 and 2 are 7
3 and 4 are 7
2 and 5 are 7

Eight

eight
8 (Arabic numeral)
VIII (Roman numeral)
8 objects
eighth (ordinal)
Eighth Grade
counting to 8
8:00 o'clock (bed time)
8:30 o'clcock
8 years old
8 steps in stairway
Eighth Street
1 nickel and three pennies are 8¢
8 dimes in 80¢
8 is 1 more than 7
8 is 2 more than half a dozen
(Subtraction facts in problems)
12 − 4 = 8 11 − 3 = 8
(Addition facts—abstract number)
4 and 4 are 8
6 and 2 are 8
5 and 3 are 8
3 and 5 are 8
2 and 6 are 8
7 and 1 are 8

Nine

nine
9 (Arabic numeral)
IX (Roman numeral)
9 objects
ninth (ordinal)
Ninth Grade
counting to 9
9:00 o'clock (school begins)
9:30 o'clock
9 years old
Ninth St. It is 2 blocks from 7th St.
9 is 1 less than 10
1 nickel and 4¢ are 9¢
1 doz. eggs, break 3—nine are left
three 3's are nine
(Subtraction facts in problems)
10 − 1 = 9 13 − 4 = 9
11 − 2 = 9
(Addition facts—abstract number)
7 and 2 are 9
1 and 8 are 9
6 and 3 are 9
5 and 4 are 9
8 and 1 are 9
4 and 5 are 9

Ten

ten
10 (Arabic number)
X (Roman numeral)
10 objects
tenth (ordinal)
counting to 10 (place in series)
counting by 10's
counting by 2's to 10
10:00 o'clock
10:30 o'clock
10 years old (in Fifth Grade)
Tenth Street
10 fingers
10 toes
10¢ in a dime
10 dimes in a dollar
10 nickels are 50¢
2 nickels are 10¢
2 candy bars for 10¢
"3 for a dime"
a 10¢ candy bar
three 3's and 1 are 10
two 3's and 4 are 10
10 is 1 more than 9
10 is 3 more than 7
10 is 5 less than 15
10 is less than 11
five 2's are 10
(Subtraction facts in problems)
12 − 2 = 10 14 − 4 = 10
(Addition facts—abstract number)
5 and 5 are 10
3 and 7 are 10
8 and 2 are 10
7 and 3 are 10
1 and 9 are 10
6 and 4 are 10
4 and 6 are 10
2 and 8 are 10
9 and 1 are 10
4 and 4 and 2 are 10

Eleven

eleven
11 (Arabic numeral)
XI (Roman numeral)
11 objects
Eleventh Grade
counting to 11
11:00 o'clock
11:30 o'clock
11 years old
Eleventh Street
11 is 1 less than 12
a dime and a penny are 11¢
(Addition facts—abstract number)
6 and 5 are 11
10 and 1 are 11
3 and 8 are 11
7 and 4 are 11
4 and 7 are 11
9 and 2 are 11

Twelve

twelve
12 (Arabic numeral)

TABLE I—*Continued*

XII (Roman numeral) 12 objects counting to 12 12:00 o'clock (noon, midnight) 12:30 o'clock 12 numbers on clock 12 hours around clock 12 hours from noon till midnight	12 years old 12 months in year 12 inches in a foot 12 shoes are 6 pairs 1 quart milk costs 12¢ 12 is one dozen three 4's are 12 four 3's are 12	(Addition facts—abstract number) 6 and 6 are 12 10 and 2 are 12 4 and 8 are 12 7 and 5 are 12 9 and 3 are 12 8 and 4 are 12 4 and 4 and 4 are 12

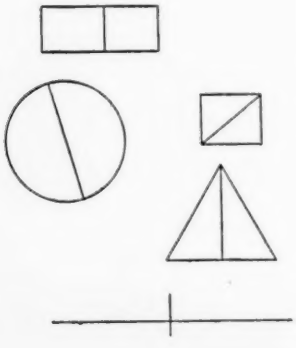
TABLE II

CONCEPTS OF FRACTIONAL PARTS

One-half

one-half
half a pie, cake, apple, etc.
half a circle
half a quart
2 halves in object (box, pie, etc.)
1 pint is half a quart
half an hour
half way around clock
half past one
half a dollar
50¢ is half a dollar
half-full
half-size
half way
half as much
half as many
half as far
half-moon
half a gallon
half a glass
half a pound
half a loaf of bread
one-half pint cream
one-half pound butter
one-half dozen is 6
6 months is half a year
one-half note in music
half of half-a-pint is ½ pint
2 is half of 4
4 is half of 8
5 is half of 10

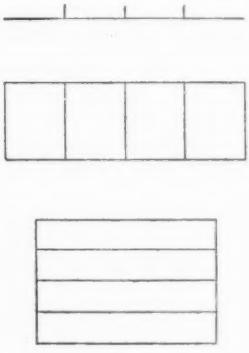
showing one-half



One-fourth

one-fourth
a quarter
4 fourths in a pie
2 quarters is half a dollar
one-fourth pound butter (cube)
one-fourth pint cream
one quarter note in music
one-fourth cup is less than ½ cup
25¢ is a quarter
4 quarters make a dollar

Showing fourths:



One-third

one-third
one-third cup

Showing thirds:

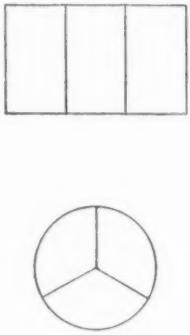


TABLE III

MISCELLANEOUS NUMBER CONCEPTS

U. S. Money

1 dime and 1 nickel are 15¢
2 dimes and 1 nickel are 25¢
3 dimes and 1 nickel are 35¢
8 dimes and 1 nickel are 85¢
10 dimes are 1 dollar
100¢ are one dollar
4 quarters are one dollar
2 half-dollars are one dollar
one-half dollar and a quarter are 75¢
one quarter and 50¢ are 75¢
3 dimes is 5 more than a quarter
2 quarters are half a dollar
8 quarters are $2.00
9 quarters are $2.25
10 quarters are $2.50
5 dimes are 50¢
2 dimes and 3 pennies are 23¢
50¢ and 20¢ are 70¢
50¢ and 50¢ are $1.00
25¢ and 5¢ are 30¢
25¢ and 25¢ are 50¢
54¢ and 10¢ are 64¢
30¢ and 2 dimes are 50¢
25¢ is a quarter
Problems:
Dorothy had 3 cents
Her daddy gave her $1.00
Then she had 103¢ or $1.03
Mary had $1.00
Her dad gave her another dollar
Then she had $2.00
Amounts of money owned by pupils

$14.00	$2.70
4.00	1.25
5.00	1.90
1.78	3.00
2.63	.78
1.40	.65

Large Numbers

7 and 7 are 14
8 and 8 are 16
four 4's are 16
10 and 7 are 17
14 and 4 are 18
9 and 9 are 18
10 and 10 are 20
two 10's are 20
11 and 11 are 22
12 and 12 are 24
10 and 10 and 5 are 25
14 and 14 are 28
15 and 15 are 30
5, 10, 15, altogether are 30
20 and 20 are 40
four 10's are 40
30 and 30 are 60
40 and 40 are 80
50 and 50 are 100
Ten 10's are 100
100 and 8 more are 108
twelve 10's are 120
100 and 100 are 200
300 and 300 are 600
1000 and 1000 are 2000
28 days in February
17 boys and girls in group
24 hours in a day
Problem:
I weigh more than Ray
He weighs 59. I weigh 60

Roman Numerals

XV	XXX
XVI	XXXIV
XVIII	XXXV
XX	C
XXI	CI
XXII	CII
XXIII	
XXIV	
XXV	

Miscellaneous

one-eighth note in music
zero (Fahrenheit) at 8:00
16 below (Fahrenheit) at 6:30
8 below (Fahrenheit) at midnight
It takes 12 years to go from First Grade through High School
Freshman is beginning college
A senior in high school is in twelfth grade
Senior High is higher than Junior High
A Junior has 1 year left of H. S.
4 ft. 5 in. is 1 ft. 5 in. more than a yard
I counted my marbles. I have 428

TABLE IV

Measurements known	Sizes of clothing		Cost of Clothing
Height	Sizes of shoes	13	$1.98 shoes
50 inches		13	7.00 coat ("quite a lot")
4 ft. 1 in.	(Child's size)	13	1.00 dress
4 ft. 2 in.		13½	
4 ft. 5 in.		13½	
4 ft. 3 in.		1	
		1	
Weight	(Grown-up's size)	1½	
61 pounds		1½	
55 pounds		2	
55 pounds	Sizes of dress	7	
60 pounds		7	
56 pounds		7½	
54 pounds		7½	
55 pounds		8	
60 pounds		10	
60 pounds			
50½ pounds	Sizes of shirts	7	
65½ pounds		8	
Between 50 and 60 pounds		9	
		10	
		10	
		10	

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An Experimental Study in Learning to Read Numerals

By LESTER R. *and* VIOLA D. WHEELER
State Teachers College, Johnson City, Tennessee

VARIOUS diagnostic studies show that errors in reading numbers are common even through high school. While frequent discussions are found concerning the abstract character of number concepts, educators have paid little attention to the difficulties involved in learning to read numerals, and have limited their investigations largely to a study of eye-movements. Dearborn (6) and Gray (8), using photographic records of eye movements, conclude: first, the time required for reading numerals increases with an increase in the digit lengths of the numerals; second, a single digit is often the unit of perception in the two-digit numerals; and third, in reading numbers the attention is concerned with smaller printed units than when reading words. According to Terry (17), when reading numerals an adult divides the material into much smaller units than when reading words, and seldom apprehends more than two numerals at a time while as many as twelve letters may be grasped at the same time. While this investigator concludes that it requires a different type of memory to keep numbers in mind from that required to recall logically coherent words, Rebert (13) suggests that numerals tend to be read in the same manner as words and that the length of pauses depends largely upon the degree to which the subject is familiar with the numerals.

In an attempt to further investigate how children learn to read numerals we have studied:

1. The efficiency of teaching the numerals 1–100 by the educational game, COUNT-O.
2. The influence of mental age on learning the numerals.
3. An analysis of the difficulties primary children encounter in learning to read numerals.
4. A comparison of learning to read words with learning to read numerals.

COUNT-O[1] is an arithmetic game designed to teach the numerals 1–100 through a free-play activity. Frequency of repetition of the numerals, vividness of presentation, typographical technicalities, length of learning time, type and quality of motivation, degree of satisfaction, variations due to individual differences, and methods of teaching are some of the factors in the learning situation which are partially controlled by the construction of the COUNT-O material. The game is simple and, even though motivated by a spirit of play, attention is focused primarily on learning the numbers rather than on the process of the game. Interest in the game increases with use—the more numbers a child learns the more successfully he plays the game.

COUNT-O is based on a flashcard technique. The game contains two sets of flashcards: one set with the numerals 1–50, and the other set from 50–100. Forty individual playing cards are provided for the children. On the side of the playing card designated "A" are found sixteen of the numerals 1 through 50, and on the reverse or "B" side sixteen of the numerals from 50 through 100. No two playing cards are alike. To play the game, the flashcards corresponding to the side of the cards the children are using are shuffled, then presented one at a time to the class. Each child scans his playing card and puts a token on the matching number if it appears on his card. The flashcard is placed in a holder in front of the class and the play is repeated. The first child to get four tokens in a row calls out "COUNT-O". The teacher and pupils

[1] Augsburg Publishing Co., Morristown, Tennessee, 1937.

then check the child for accuracy by having him read aloud the four numbers in the row he has completed. If they all check with the flashcards displayed in front of the class, the child wins the game. As it takes only a short time to run out one game, there is opportunity for several children to win during one play period.

COUNT-O differs from Bingo, Lotto, and similar games used occasionally by primary teachers in that it has been standardized on first grade children and has been carefully designed for teaching. All the numerals through 100 are included; there are no call-letters to confuse the child; and, by dividing the numerals 1–100 into two games, learning is simplified. The size and style of type is the same as cards were well shuffled before each testing.

Immediately after this initial testing the game of COUNT-O was introduced to the first-graders. During the following five days the teachers played COUNT-O twenty minutes a day, then the individual testing was repeated in the same manner as for the initial test. No incentive other than the game was used for motivation, and the teachers were not assisted in any way other than to administer the tests and instruct them in the use of the game.

Table I gives the results of Test I in Grades I and II, and the difference indicates the progress normally made during the first year when the numerals 1–100 are taught by the usual classroom meth-

TABLE I
RESULTS OF INITIAL TESTING

Grade	Number of Cases	Median I.Q.	Median Score	Range	Q − 1	Q − 3	Q
I	157	98 ± 1.25	9.21 ± .85	0–100	.99	24.58	11.80
II	103	98 ± .93	96.51 ± .74	23–100	87.90	99.91	6.01
Difference			87.30 ± 1.12				

that found in the child's books and other school materials, thus aiding transfer by reducing perceptual difficulties.[2]

Data for this experiment were gathered during five consecutive school days in November, from first and second grade children in the public schools of Johnson City, Tennessee. One hundred fifty-seven first grade children and 103 second grade children were tested individually with the COUNT-O flashcards for immediate recognition of the numerals 1–100. Individual records were kept of the numerals each child knew and the types of errors made. All testing was carefully done by students especially trained and experienced in this method of testing. Good rapport was established with each child before testing, and the child proceeded to read the numbers at his own pace. A game was made of the testing by giving the child the cards he read correctly. All ods. Although we cannot say at what rate the average child learns to read numbers during this period, it is evident that he knows them fairly well by the time he reaches second grade. There appears a wide range in individual differences; in Grade I about eight per cent knew all the numbers, 19 per cent knew ninety or more, while 12 per cent knew less than ten. In Grade II there is also a surprisingly wide range between the poorest and the best pupils.

Of the 157 cases tested in Grade I, thirty-one children who knew ninety or more of the hundred numerals were eliminated from the experiment in order to get a more accurate measure of the rate of learning, and twelve more cases were dropped because of absences during the experiment, leaving 114 children receiving the final test. The low correlation between I.Q. and learning the numerals shows that dropping these cases did not materially affect the general intelligence level of the remaining group.

[2] For an evaluation of Educational Games, see: Minor, Ruby. Arithmetic Games, Educational Method, January 1937, p. 211.

Table II shows that the experimental group knew about six numerals at the beginning of the experiment. This increased to sixty on the second test, showing a gain of 53.71 ± 4.32 numerals during five days, or a gain of 10.74 numerals a day. The statistical significance of this gain is indicated by the *D/P.E.* diff. and by the first and third quartiles. The individual differences in ability to learn to read numerals are shown by the range and the increase in the semi-interquartile range.

How does the rate of learning to read numbers compare with the rate of learning to read new words? An experiment made by Wheeler (21) shows that children in Grade IB learn about .84 of a word a day when playing READ-O twenty minutes a day. Since READ-O is subject to controls similar to those of COUNT-O, the principal difference occuring in the substitution of the first seventy-five words in Gates' Word List for the numerals in the COUNT-O game, a fairly accurate comparison may be drawn from these two experiments. When equal opportunity is given, first grade children learn to read numerals over ten times faster than they learn the most frequently used words. In other words, the first grade child learns to read numerals much easier than words. Various investigations of number readiness have shown that the idea of number is present at an early age. Possibly this fact, in addition to the simplicity of the configuration of number symbols compared with that of words, may help to explain the ease with which first-graders learn the numerals.

What effect does mental maturity have on children's ability to read numbers? Buckingham and MacLatchy (3) found that children use numbers when they enter the first grade. Washburne (20) considers the minimum mental age of 6 years—5 months advisable before a child undertakes to learn the addition facts with sums of 10 and under, and a minmum mental age of 7 years-4 months for the addition facts with sums over 10. Since perception and recognition of the numerals are prerequisites, one assumes that the average child should read numerals at the mental age of near six-and-a-half or seven years.

Table III shows that while the median mental age among the children who learned 75 or more of the hundred numerals was 7 years-2 months, there was a wide range in M.A.'s from 4 years-11 months to 9 years-10 months, and a similar trend is indicated throughout the table. The correlations between mental age and learning numerals are:

M.A. with Test I, r $.51 \pm .056$

M.A. with Test II, $r = .43 \pm .061$

While these correlations show a fairly substantial relationship, they suggest that

TABLE II

RESULTS OF TESTS I AND II, SHOWING GAIN IN READING NUMERALS DURING FIVE DAYS TEACHING BY COUNT-O

Test	Number of Cases	Median Score	Range	Q − 1	Q − 3	Q
I	114	$6.29 \pm .83$	0– 87	1.17	15.36	7.10
II	114	60.00 ± 4.24	2–100	18.61	91.56	36.48
Difference D		53.71 ± 4.32				
P.E. diff.		12.43				
Chances in 100		100				

TABLE III

RELATION OF MENTAL MATURITY TO LEARNING NUMERALS

Number of Numerals Learned During the Experiment	Median M.A. Yrs.-Mos.	Range of M.A.'s Yrs.-Mos.		Yrs.-Mos.
75–100	7-2	4-11	to	9-10
50– 75	6-3	5- 3	to	8- 5
25– 50	6-6	5- 5	to	7- 4
0– 25	6-1	5- 0	to	6- 9

TABLE IV

PERCENTAGE OF CHILDREN WHO READ EACH NUMERAL CORRECTLY ON TESTS I AND II, AND GAIN DURING FIVE DAYS OF EXPERIMENT, GRADE I

No.	Per Cent Correct				Gain	Av.	No.	Per Cent Correct				Gain	Av.
	Test I	Av.	Test II	Av.				Test I	Av.	Test II	Av.		
1	86		99		13		50	6		45		39	
2	69		85		16		51	6		53		47	
3	79		89		10		52	6		54		48	
4	68		85		17		53	9		52		43	
5	74		81		7		54	8		54		46	
6	57		70		13		55	6		55		49	
7	54		74		20		56	6		50		44	
8	60		82		22		57	7		50		43	
9	33	64	67	82	34	17	58	4		55		51	
							59	5	6	46	51	41	42
10	45		50		5		60	11		54		43	
11	30		46		16		61	11		66		55	
12	10		20		10		62	14		59		45	
13	10		27		17		63	12		60		48	
14	11		30		19		64	11		63		52	
15	11		25		14		65	13		60		47	
16	8		30		22		66	14		63		49	
17	11		31		20		67	12		59		47	
18	11		31		20		68	11		60		49	
19	9	15	30	32	21	16	69	11	12	56	60	45	49
20	16		38		22		70	11		65		54	
21	13		45		32		71	14		67		53	
22	16		46		30		72	15		64		49	
23	13		39		26		73	14		65		51	
24	13		37		24		74	13		62		49	
25	12		42		30		75	16		64		48	
26	13		39		26		76	11		57		46	
27	10		39		29		77	13		66		53	
28	11		38		27		78	13		65		52	
29	7	13	30	44	23	27	79	11	13	59	57	48	50
30	7		48		41		80	10		61		51	
31	11		52		41		81	12		68		56	
32	9		50		41		82	11		63		52	
33	14		50		36		83	14		69		55	
34	10		55		45		84	15		68		53	
35	10		49		39		85	15		67		52	
36	4		45		41		86	13		58		45	
37	5		41		36		87	12		63		51	
38	9		46		37		88	14		67		53	
39	3	8	41	48	38	40	89	11	13	60	64	49	52
40	11		54		43		90	9		54		45	
41	14		67		53		91	11		63		52	
42	13		65		52		92	10		61		51	
43	14		58		44		93	11		65		54	
44	14		60		46		94	10		60		50	
45	12		66		54		95	11		62		51	
46	10		57		47		96	8		55		47	
47	11		58		47		97	7		56		49	
48	11		63		52		98	13		60		47	
49	11	12	49	60	38	48	99	9	10	59	60	50	56
							100	25		92		67	

mental age is only one of many factors influencing ability to read numerals, and they are too low to justify any conclusion that mental age is an adequate criterion for curriculum placement of subject matter—a theory widely advocated by several educators. According to Hull's formula,[3] prognosis of success on the basis of mental age is likely to be dependable in less than thirteen per cent of the cases. The correlation of I.Q.'s with rate of learning nu-

[3] Hull, C. L. The Correlation Coefficient and Its Prognostic Significance, Journal of Educational Research, May 1927, 15: 327–339.

merals also shows a low relationship, $r = .30 \pm .008$. Individual differences in habits of study, interests, physical maturation, motivation, environmental experiences, and probably many other factors influence the achievement of children, and are important in determining the place of a subject in the curriculum. Our findings substantiate the theoretical opinion given by Brownell in his criticism of the Committee of Seven's Investigations.[4]

Table IV shows the percentage of children reading each numeral correctly on Tests I and II, and the amount of gain for each numeral during the five days of the experiment. Since the children had received some instruction prior to the experiment on number concepts and reading numbers 1–10, it was expected a large percentage of the children were familiar with the digits. We find that an average of 64 per cent knew the digits on the first test. This represents the amount they had learned during approximately three months of school, and within five days it was raised by the COUNT-O game to 82 per cent. Apparently 9 is the most difficult digit to learn; 8 is evidently less confusing than 6, 7 or 9; and 2 and 4 are more difficult than 3 and 1. Because so many of the children were familiar with the digits at the beginning of the experiment, the small gain made here is not comparable to that made with the rest of the numerals.

It is evident from Table IV that the "teens," as a group, are the hardest. Of these, the numeral 12 appears to give the most difficulty. The "teens" also cause the greatest difficulty in the second grade. When we eliminate the digits, the numerals are ranked in order of difficulty from easiest to hardest as follows:

Rank	1	3	3	3	5	6	7	8	9
Numerals grouped by decades	80's	40's	60's	90's	70's	50's	30's	20's	"teens"

The numeral 100 is evidently as easy for the child to learn as 1; these two numerals were the only ones that over 90 per cent of the children recognized on the second test. In general, the larger numerals are learned more rapidly than the smaller. The average gain made during the experiment consistently increases with an increase in the size of the numerals. The average of the difficulty ranks for the digits, "teens," twenties, thirties, etc., correlated with the average gain for each of these groups shows a close relationship, $r = .96 \pm .016$. In general the difficulty of learning to read the numerals decreases with an increase in the size of the numeral. Combining reasoning with memory during the learning process, by naming the numerals from the logical order of digits making up the numerals, may explain this trend.

An analysis of the types of errors made by first and second grade children shows that the two major difficulties in reading numerals involve reversal tendencies and confused perceptual patterns. Children have a strong tendency, especially in reading the "teens," to reverse the digits, calling 13, 31 and 15, 51 etc. It was observed that while the first attack or reaction might be to reverse the digits, the child often caught his mistake before vocalizing the entire numeral. While there is a marked tendency toward reversing other numerals, this error was most pronounced in respect to the "teens" and any number containing 1. Besides reversals, there was a marked confusion due to inversions such as occur when reading 6 for 9, or 9 for 6. Investigations of eye movements show that a single digit is usually the unit of perception in the two-digit numerals. Both reversals and inversions of numerals depend largely on the physical maturation of the eye involving vertical and lateral imbalance, and special training might prevent or eliminate these difficulties.

Other errors, due to imperfect percep-

[4] Brownell, W. A. A Critique of the Committee of Seven's Investigations, Elementary School Journal, Mar. 1938, 38: 495–509.

tion of the form of the digits, commonly occur in confusing 7 and 9, 8 and 3, and occasionally 5 is confused with 7 and 3. Undoubtedly the general contour of the figures explains these perceptual errors. The child catches the vague, indefinite, total configuration without attending to the precise details. In case of confusing 5, 3 and 7, a reversal in the top line of 5 results in similar focusing points suggesting a 7 or a 3. Similar errors are found in the perception of letter symbols; b is often confused with d, t mistaken for l, m for w, e for o, etc. Gates[5] explains that such errors are due to lack of practice in letter recognition rather than reversals or perceptual habits. If this were so, in reading numerals one would expect that familiarity with the digits would eliminate the tendency to invert or reverse the numbers. This does not seem to be the case; in this investigation children who knew the digits well often showed a tendency toward reversal and perceptual errors. While it is obvious that children must have many experiences with numbers or letters in order to overcome these errors, it seems very probable that there are factors causing these difficulties other than unfamiliarity. A further comparative study is being made to investigate the factors affecting reversals in letters and digits, words and numerals.

Several pedagogical implications are suggested in the results of this investigation. A ranking of the relative difficulty of the numerals offers the primary teacher the means for improving her efficiency in teaching children to read numers. By knowing what numbers or groups of numbers are likely to cause the most trouble, she can better plan her teaching to eliminate or overcome those difficulties and apply preventive rather than remedial measures. If the teacher realizes also the common perceptual difficulties her children are likely to encounter, she can plan drill and teaching techniques especially designed to overcome these errors.

[5] Gates, A. I. The Improvement of Reading, Macmillan, 1937, pp. 355–356.

Since children develop interest in number at an early age and learn to read numerals much more easily than they learn to read words, beginning instruction in the first grade with numbers rather than words may be an economical teaching procedure. Numberland—so full of everyday objects to count, games, the calendar, clock, speedometer, telephone, house and automobile numbers—is a fascinating world for the young child. There is no easier method of integrating school with life than through number experiences. Building number concepts need not necessarily precede teaching the number symbols, but may be taught successfully at the same time. Too rigid adherence to teaching concepts definitely impedes the progress of first-graders who are interested in and have a need for numbers above 10. It is awkward and impractical to attempt to teach concepts beyond the digits by the methods advocated by Brownell and others. For example, try to teach by these methods the concept of 67 or 100 to a first grade child, or 1,000,000 to an eighth-grader, yet both children may need these numbers to intelligently understand their environment.

Because eye movements are slower, the child may read numerals more accurately than words. To read accurately, he needs training in correct perceptual habits, and such training with numerals might help to overcome reversal tendencies and poor habits of eye movements when the child begins reading words. When the child reads words, often he needs to see only certain clues in order to recognize a word, and it is not advisable to slow the reading process by too careful analysis of the details of words. Perhaps learning to read numerals, where accuracy in perception is necessary, desirable, and in no way conflicts with other reading factors, offers fertile ground for habit formation before the child is launched into the difficulties of a reading program. Arithmetic, rather than reading, may be a more economical starting point for the first-grader.

The rapid progress made by children

in this experiment suggests that indirect methods of teaching primary children have considerable value. Similar results have been found in teaching the addition combinations (22) and in the field of reading (21). From experimental data presented, it is apparent that within ten days the first grade child may easily learn to recognize and read all the numerals 1–100 without any instruction other than that hidden in a fascinating game. Indirect advertising is most profitable for business, and it seems highly probable that similar methods might prove equally successful in the classroom.

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Extra-Curricular Mathematical Activities In Secondary Schools

By Sister M. Leontius Schulte, O.S.F.
College of Saint Teresa, Winona, Minnesota

It has long been the practice of the writer to advocate extra-curricular activities as an important supplementary means for stimulating the interest of pupils in secondary courses in mathematics. It is true that the development and maintenance of interest in mathematics depends primarily on the presentation of the subject by the teacher, but it often becomes necessary to use additional devices for promoting enthusiasm. Extra-curricular activities in mathematics, including those of a recreational character, are invaluable for vitalizing the so-called dead language of mathematics.

A great number of pupils have an aversion to mathematics. Something in the nature of an extra-curricular activity will often change this attitude of mind. An improved attitude can actually be brought about by means of project work along with the regular class work. It is not at all unusual to find a pupil who approaches geometry with a critical attitude and later revels in it, in response to some creative work suggested by the teacher. Once the student has the proper feeling towards mathematics, its greater difficulties are surmounted. If any project arouses sufficient enthusiasm to achieve these results, this alone warrants its execution.

Many teachers of mathematics in high school are at a loss when there is question of finding methods for making the usual topics of algebra and geometry appeal to the pupils. Perhaps these teachers never actually participated in extra-curricular activities while in high school or college. It was a practical suggestion given the writer by her instructor of geometry in high school which made her later, as a teacher in high school, carry out a plan for project work. The pupils responded generously to a call for contributions of geometric designs made with compasses and ruler. Following another practical suggestion, the theorems on similar triangles became more real by means of actual field work in geometry. In a similar way, these ideas can be used to great advantage by making it part of a teachers' course to train future teachers of mathematics in high school to carry out projects of the same nature.

During the second half of the year, high school pupils can be asked to work on projects of a recreational character having a bearing on the algebra and geometry studied in the first part of the year. General suggestions can be made as to the types of projects suitable for work of this nature. The contributions should be purely voluntary, and should be made merely for the joy derived therefrom. No promises of remuneration need be made; but because of splendid cooperation and amazing results, prizes may be awarded. All who take part will be fully convinced that mathematics can be a fascinating recreational activity. Every project submitted should be put on display so that each pupil can see what the others have done. This not only arouses the competitive enthusiasm of each individual pupil who takes an active part, but it also arouses new interest in the minds of those who have hitherto looked upon mathematics with disfavor.

The variety of projects which may be submitted is amazing. Teachers of high school mathematics may perhaps welcome these suggestions for projects of the various types:

1. *Literary contributions.* A wide choice may be presented here. There may be original poems of a mathematical nature, and parodies of poems, such as Kilmer's *Trees* and Kipling's *L'Envoi.* In some

instances mathematical thoughts can be adapted to famous literary selections, Mark Antony's Speech, Macbeth's Speech, and the Gettysburg Address. Many popular songs lend themselves to algebraic and geometric ideas. Other papers may take the form of short stories. To illustrate, one, entitled *The Naughty Little Three*, could be the story of a naughty little three who got into the denominator under a radical sign and could not escape until it became rationalized. Mathematical plays, such as a dialogue between the decimal point and zero, may form very instructive contributions. There may also be mathematical alphabets, theorems, news items, newspapers, and essays written on numbers.

2. *Instructive devices and popular recreations.* Great ingenuity may be displayed in the various cross-word puzzles submitted. Some may result in scramblegrams, wherein the given letters are to be unscrambled to form a mathematical word whose definition is given. A missing-word story would aid greatly in developing a vocabulary of geometric terms, and would appeal to students of geometry especially if it bore such a title as: *The Sad Geometrical Romance of Polly Gon.* The once-popular jig-saw puzzle finds an occasional devotee who subjects some complicated geometric design to the jig-saw. The special products and types of factoring problems in algebra lend themselves very well to a mathematical wheel.

Games of divers kinds may be improvised: a mathematical game of golf, the once-popular game of imp, and a game of cards. The latter project could consist in matching important formulas according to rules similar to the rules used in "Authors." It would serve the purpose of familiarizing pupils with the outstanding facts of algebra, and could be used to great advantage in high school as a less monotonous substitute for the ordinary drills on special products and factoring methods. A simple factoring method may be presented in the form of a guessing game in the following manner:

Think of a number.

Square it.

Subtract 25 (or any square number).

Divide by the number plus 5 (or plus the square root of the number subtracted).

To the result add 5.

After a short time the participants will realize that the correct guess depend upon this statement:

$$\frac{x^2 - y^2}{x + y} = x - y$$

3. *Drawings.* Drawings made with straight lines, with curves, and with the digits could be submitted. Posters and booklets on mathematical subjects, cartoons, mottoes, and graphs of data of local school interests present much variety.

Project work can be justified not only for its value as an incentive to high school pupils to the further pursuance of mathematics, and for its service to future teachers of mathematics in secondary schools, but also for its value as a potent factor in making the Mathematics Club a vital, active organization in any school.

Very frequently the Mathematics Club in high school needs to be awakened to renewed activity. Since an occasional evening of a social nature very often kindles the spark of enthusiasm, a progressive party may be arranged. The entertainment can center around some of the projects mentioned above. At no time need the interest of the members wane; in fact, the time allotted for these recreations will seem altogether too short. After the awarding of prizes, each guest will draw a simple equation, the solution of which locates her place in the tea room. The favor may consist of a paper tetrahedron containing scientific nuts in the form of quotations and tid-bits which will serve in the future as food for thought. On the face of the figure may be printed a menu containing mathematical terms. Many of the literary contributions described above, together with mathematical jokes, may be utilized for toasts.

Extra-curricular activities of the kind described could profitably be begun in elementary school and carried on in high school. A judicious use of these or of similar activities in high school would undoubtedly stimulate a greater enthusiasm for mathematics, and would open to the pupil an avenue of new possibilities of interest in the field of higher mathematics. If pupils in high school would be actuated by the proper mental attitude toward mathematics, how much real mathematical work could be achieved in comparison to what is now accomplished!

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The Use of Calculating Machines in Teaching Arithmetic

By W. S. Schlauch
New York University

Calculating machines were invented, and are used in business, industry and finance as well as in research and statistics to accomplish the following objectives:

1. To lessen the labor of computation.
2. To increase the speed with which computations are made.
3. To guarantee the accuracy of the results of computation.

These objectives have been attained, and the use of these machines has become wide spread. Almost any retail business employs a cash register, which is an adding machine. Banks and trust companies employ adding machines. The statistical departments of our large corporations are equipped with electrically operated machines which add, subtract, multiply, and divide with amazing rapidity; and, if the proper digit keys have been depressed, with absolute accuracy. Scientific laboratories and engineers' offices find their help indispensable.

Since education is preparation for complete living, school experiences being a preparation for life in the world, the question arises: Should the pupils be taught to perform the computations arising in the solution of problems in arithmetic by the use of machines, as they would in a similar situation in real life?

The answer to this question depends on several factors, the principal ones being the collateral educational benefit accruing from the use of such machines, and their cost. Not many years ago typewriters were found only in favored schools in wealthy districts, and they are now common equipment. Calculating machines at a moderate cost, hand operated, which perform the four fundamental operations rapidly and silently have been developed by at least one calculating machine company,[1] and other companies probably have similar machines. Their cost is little more than that of a good typewriter, and almost any high school can be equipped with a sufficient number of them to enable each pupil of a class to operate one under the direction of a teacher.

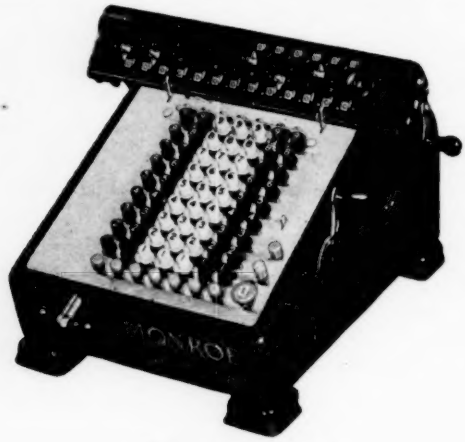
About twelve years ago, when I was Chairman of the department of mathematics in the High School of Commerce, in New York City, we conducted an experiment in the use of calculating machines in a review course in business arithmetic, taught in the second semester of the senior year. The object of this review course was to raise the level of efficiency of our graduating class in business calculations just before entering the business world. These students had studied business mathematics, using algebraic methods during their four year course, but it was considered desirable to "freshen up" their business arithmetic just before graduation, since the first positions they would occupy would call for routine calculations or computations in arithmetic. The senior classes had never evinced any enthusiasm for this review course, since they had had arithmetic in some form during six school years of their careers.

When the Monroe Calculating Machine Company agreed to lend us forty machines as an equipment in teaching this review course in business arithmetic, the machine replacing pad and pencil for computation work, there was an immediate change of attitude toward the arithmetic course on the part of the seniors. They looked forward to the arithmetic period as a treat, and were reluctant to stop work at the end of the period. The technique of op-

[1] The machine referred to is the Monroe Calculating Machine Company's "Educator."

erating the machines was mastered in ten periods of the semester including addition, subtraction, multiplication, and division. Group instruction in operating the machines was given during the first part of the period, and individual practice by the pupils with help from the teacher followed during the second half, making the controls learned automatic and reliable. When the machines were not in class use, they were locked in the large closet set apart for them.

For school men the important question is: Did the use of the machines help the



MONROE EDUCATOR
ADDING-CALCULATOR

students to learn business arithmetic? The answer to this question may be broken up into several phases.

1. The introduction of the machines stimulated interest in performing the fundamental operations rapidly and accurately. Pupils were taught the regular method of performing these operations, and were allowed to work out short cuts for themselves. Thus, after learning to multiply a number by 98 by turning the crank forward with the repeat key depressed, 8 times, shifting the carriage one place to the right, and then turning the crank forward 9 times, nearly every student discovered for himself that the same result can be obtained by *three* motions of the crank: multiply by 100, and subtract two times the number. Discovering number relations was a valuable by-product, but arousing interest in an old subject was the first great benefit conferred by use of the machines.

2. The speed with which computations could be made by the pupils, with the machines, after a few weeks of practice was so much greater than by the pad and pencil method, that a greater number of problems under each heading or category could be solved in a class period. This helped to fix the principle or method of analysis developed.

3. Checks of the accuracy of the results of the solution of problems could be easily applied with the help of the machines. Checking results became a habit on the part of the pupils, to a greater extent than had been the case under the pad and pencil method.

4. Pupils were encouraged to use the most efficient method for the solution of a problem, i.e., for performing the computations involved, whether pad and pencil, or machine. Thus they learned that to discount a note whose maturity value was \$815.85 for 128 days at $4\frac{1}{2}\%$, mental process combined with pad and pencil is more efficient than use of the machine; but summing the debit and credit columns of a trial balance is more easily done with the machine.

5. While no attempt was made to select two equivalent groups, as to I.Q., mental age etc., and use one group as the machine experimental group, and the other as a control group, our experience seemed to show that the ability to solve problems that call for analysis, was increased by the term's work about as much without the use of machines as with them. The median score of a class in a test made up of such problems at the end of the semester, as compared with the median score on an equivalent test at the beginning of the semester, tended to show that about the same percent of improvement was made during the three semesters of the experiment as was experienced before the introduction of the

machines. The percent improvement in *computation* speed and accuracy, on the other hand was much greater with the use of the machines than without them.

Since the experiment in the High School of Commerce,[2] the use of machines in teaching arithmetic has been tried in a number of schools. These include the James Monroe Evening High School, New York; the Benjamin Franklin (Junior), High School, New York; the Belmont Junior High School, Dayton, Ohio; the Lamond School, Shaker Heights, Cleveland, Ohio; with sixth grade pupils; and the Morristown High School, Morristown, New Jersey.

HIGH SCHOOL OF COMMERCE

The testimony of the men making the experiments in these schools is well worth reading in full. Through practically all of their reports there appears the following testimony as to the value of calculating machine use in teaching arithmetic:

1. They are a strong motivating force. They arouse interest in arithmetic because pupils like to use the machines, and because they relieve them of the drudgery of computation.

2. In problem solving, the analytic or thought processes were just as prominent as with the pad and pencil method. Before a pupil could put a problem into the machine, he had to analyze it in advance.

3. The routine calculations or computations necessary to complete the solution of a problem after the analysis was completed, were performed so quickly with the help of the machines that more analyses of problems could be made in a given period than by the pad and pencil method.

4. Typical instructional material in the sixth grade does not provide enough problem drill to justify extensive use of calculating machines in this grade. They are used to best advantage in the senior high school or the upper grades of the junior high school.

5. The use of machines makes easy, and gives insight into certain number rela-

[2] The High School of Commerce now owns more than 20 such machines and the Trenton (N.J.) Senior High School owns 35. These schools find them very helpful in teaching arithmetic.

tions, such as transforming fractions into decimals locating the decimal point in the result of computations, the fact that multiplication is repeated addition and division is repeated subtraction, etc.

Perhaps the most significant experiment in the use of calculating machines in the teaching of arithmetic that has come to my attention is that initiated and supervised by Francis Grant Marsh, in working out a thesis for his Doctorate in the University of California. This thesis is entitled: "An Experimental Study of the Facilitation and Interference Effects of Calculating Machines upon Arithmetical Skills."

Three schools in the San Francisco area, two being four year high schools, and one a junior high school, were used. Two groups of students in each school were selected; one, the machine group (E), and the other the control group (C). After the classes were organized, tests were administered to ascertain the mental and arithmetic abilities of the pupils. From the results of these tests it was possible to equate two groups on the bases of mental age and arithmetical ability, in addition to sex and grade level. The final comparisons were made between two very nearly equal groups, each composed of 38 boys and 36 girls, or a total of 152 pupils. The experiment was made during the spring semester of the school year, 1933–34.

In the summary of findings, Dr. Marsh states that substantial and almost equal gains were made in arithmetical ability by the machine group and by the control group. The slight difference favored the control group. Since the same teacher taught both groups in all the schools in the same way in this phase of the subject, and since computation is decidedly subsidiary to analytic thought process in arithmetical reasoning, this result is not surprising.

In the final test in "Business Arithmetic" the mean scores of the two groups were approximately equal: "The small difference of 0.58 between the mean scores, favoring the E group was found to be not statistically significant."

The findings in determining the abilities of the groups to *compute* arithmetical exercises show that a difference of mean scores of 4.66 existed in favor of the C group at the beginning of the experiment, and that a difference of 12.83 between the mean scores on the final test favored the E (machine) group. Thus the greater gain in ability to compute was made by the machine group.

The teachers who taught these experimental groups were of the opinion that the machine groups showed much interest in their work, began the assignment without delay, and worked diligently throughout the period. They also testified that the classroom management and procedure were less difficult in the machine groups than in the control groups.

We may conclude that the use of calculating machines in teaching arithmetic is justified by its results, and that they should be used wherever the cost of installing such machines can be met. They are used to best advantage in the upper grades of the junior high school and in the senior high school. They lend speed, accuracy, and confidence in computation, and stimulate an interest in problem solving calling for their use.

HAVE YOU seen the program of the twenty-first annual meeting of the National Council of Teachers of Mathematics at St. Louis on February 22 and 23 on page 41?

The names of the hosts and hostesses, the subjects which they are to discuss, and means for reservations will be published in the February issue of *The Mathematics Teacher*.

◆ THE ART OF TEACHING ◆

A Helpful Technique in Teaching Solid Geometry

By JAMES V. BERNARDO
High School, Plainville, Conn.

IT HAS BEEN my experience, as it has been undoubtedly that of many who teach solid geometry, to find that the three-dimensional concepts are not easily conceived by the average student. He does not comprehend fully the meaning of the drawings of "solid" figures in one plane. To develop an aptitude for drawing and for interpreting figures is the real job for the teacher who is attempting to expound the propositions of the sixth, seventh, and eighth "Books" of Euclid.

Strange to say, there is not, comparatively, a very great wealth of material written on the teaching of solid geometry. I have read as much as is available to me on the subject; but not being satisfied I have undertaken a project which is largely experimental and which has shown splendid results thus far.

The first unit of work is actually exploratory from the standpoint of the teacher and of the pupils. Each pupil is asked to make drawings of various "solid" objects in the classroom. The degree of proficiency and accuracy in representing these objects by a drawing in the plane of the paper proves to be a trying experience for many of them. Only those who have had mechanical-drawing or an art course show, as a group, adeptness. Therefore, the teacher learns from these papers the abilities of each individual in this department of the work. A short course in the methods of drawing then follows.

After this preliminary study of geometric representations in three dimensions, the class is launched into Book V. But at this point a deviation from the ordinary procedures is made. One theorem is assigned to each pupil in advance. Let us take pupil A for example. He is assigned to theorem II which reads, "If two planes intersect, their intersection is a straight line." His assignment requires that he make a model of the "figure" and then demonstrate the proposition to the class, using a figure drawn on the board together with the model. Each step written on the board is thus "seen" to be true from the actual model of the set-up. After pupil A has "taught" his proposition, the class raises any discussion necessary for a complete understanding of the proposition. Much interest results from questions "flung" at the demonstrator, for there are many special cases to be treated in a large number of the theorems.

The one-to-one correspondence set up between the drawing on the blackboard and the model in hand, helps the individual to correlate the physical actuality with the hypothetical figure in the plane.

Upon completion of the treatment of the proposition, the model is put upon a table in the front of the class-room. Hence, there is a model for each theorem studied. The pupils refer to this table as a history student refers to the library. That is, to facilitate the drawing of figures, a pupil may make use of any of the models.

As to the making of the models, let us again refer to pupil A, mentioned above. With two pieces of 3/32″ white cardboard he can easily construct two intersecting planes by cutting a slot in each piece, and then fitting them together. The intersection is then painted, or colored with

crayon to accentuate the "line of intersection" to be dealt with in the theorem. Points are made by large black dots, and are labeled with heavy black letters. i.e. Figures below:

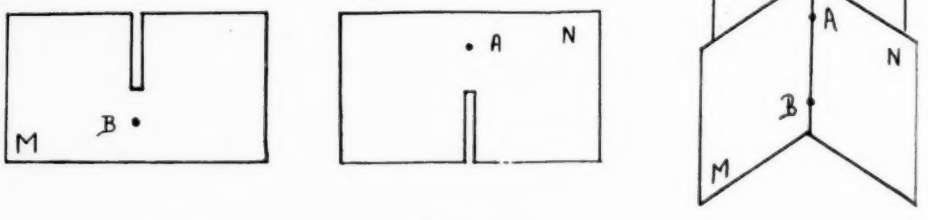


FIGURE 1

For theorems involving lines perpendicular to planes the pupils have been very ingenious. Lolly pop sticks, for instance are used to great advantage in a theorem such as "Construct a plane containing a given point and perpendicular to a given line." A piece of cardboard cut into a rectangle with a hole at P, and two straight lines such as PR and PS, together with the lollypop stick through P, gives a very desirable and simple model. i.e. Figures:

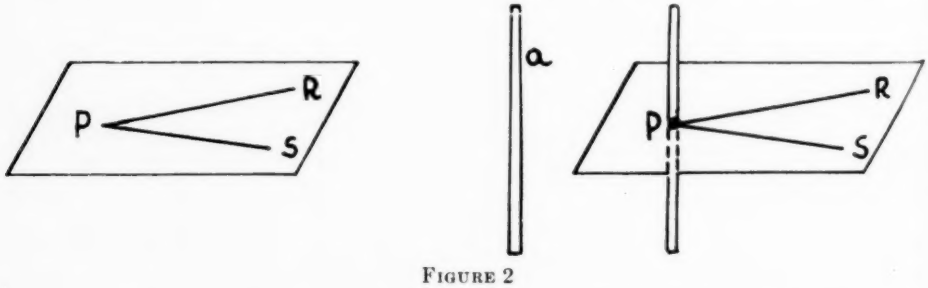


FIGURE 2

For more complicated theorems such as "If a line is perpendicular to two lines at their point of intersection, it is perpendicular to any line in their plane through that point," a "string" model can be made. Some of my students turned in excellent models made with cardboard, some sticks, and heavy black cord.

When the unit on polyhedrons, cones, and cylinders is studied, the models can be made of wood or cardboard. The five regular solids, namely, tetrahedron, cube, octahedron, dodecahedron, and icosahedron are required of all the students, and I may say with surety that they like the work. It is a great satisfaction to them to see these objects with such awe-inspiring Greek names take form before their eyes, by use of their own hands, and with their own knowledge.

We all know the truth of the saying, "to teach is to learn." Hence, by attempting to really convince the class of the truth of his assigned proposition, a pupil really witnesses that truly great feeling resulting from teaching. Moreover, the student seems to receive the assignment as an opportunity, and not as a task. The greater the amount of purposeful activity put into the hands of the class, the better and more abundant responses and total results. Finally, then, I conclude with the expression of my belief that this method of "instruction-by-the-student" has great advantages and tremendous possibilities. It has been highly satisfactory and interesting in my experiences.

Program, National Council of Teachers of Mathematics

Hotel Chase, St. Louis, February 22-23, 1940

Theme: Mathematics for the Other-than-College-Preparatory Student

I. General Meetings

1. February 22—4:30–5:30 p.m. Get-acquainted and see-exhibits hour. Reception committee in charge.
2. February 22—8:00 p.m. Presiding: H. C. Christofferson. Address of Welcome, John Rush Powell, Assistant superintendent of St. Louis Schools. Modern Youth Challenge the Curriculum, J. Paul Leonard, Stanford University. Mathematics in Education as Viewed by a School Administrator, John L. Bracken, Superintendent of Schools, Clayton, Mo. Mathematics in and for the Modern Curriculum, W. D. Reeve, Columbia University, Teachers College.
3. February 23—8:30–10:00 a.m. Annual Business Meeting.
4. February 23—12:15–3:00 p.m. Luncheon for Representatives, Delegates, and Directors. A. E. Katra in charge.
5. February 23—1:00–3:00 p.m. Visual Aids. Presiding: E. R. Breslich, University of Chicago. The Use of Films in Motivating the Study of Mathematics, The Chevrolet Motor Co. The Use of Slides and Films in Correlating Mathematics with Other Fields, Ida Fogelson, Bowen High School, Chicago. The Use of Films in Supplementing the Teaching of Mathematics. Note: The Chicago Model Club and the Schools of St. Louis and vicinity have on display Thursday and Friday an exhibit of varied visual aids.
6. February 23—6:00–8:00 p.m. Discussion Banquet. 8:00–9:00 p.m. Program. Music—Glee Club, John Burroughs School, Ralph Weinrich, Director. Facts and Fancy, C. A. Hutchinson, University of Colorado, Boulder.

II. Elementary Schools Programs

1. February 22—10:15–11:45 a.m. Presiding: C. L. Thiele, Detroit, Michigan. Analytic Analyses in Arithmetic Processes, Irene Sauble, Supervisor of Exact Sciences, Detroit. Discussion Leader, Helen Turley, Weiman School, St. Louis.
2. February 22—2:00–4:00 p.m. Presiding: Foster Grossnickle, State Normal School, Jersey City. Number Readiness, Josephine MacLatchy, Bureau of Educational Research, Ohio State University, Columbus. Discussion Leader, Marguerite Versen, Primary Supervisor of St. Louis Schools.
3. February 23—10:00–11:45 a.m. Presiding: Jesse Osborn, Harris Teachers College, St. Louis. The Nature and Purpose of Consumer Mathematics, H. R. Risinger, School of Education, Rutgers University. Discussion Leader, Jesse Osborn.
4. February 23—3:00–5:00 p.m. Presiding: C. L. Thiele, Detroit, Michigan. Psychology and Its Arithmetic Applications, T. R. McConnell, Professor of Education and Chairman of Commission on Educational Research, University of Minnesota, Minneapolis. Discussion Leaders, Stephen Gribble, Washington University and George R. Johnston, Director of Tests and Measurements in the St. Louis Schools.

III. Secondary Schools Programs

1. February 22—8:30–10:00 a.m. Mathematics for the High School Pupil Not Going to College. Presiding: Mary A. Potter, Racine, Wisconsin, Who Is the Non-college Preparatory Pupil?, Virgil Mallory, State Teachers College, Montclair, New Jersey.
2. February 22—10:15–11:15 a.m. Consumer Education and Mathematics in the Junior and Senior High Schools. Presiding: Dr. J. Paul Leonard, School of Education, Stanford University. Mathematics in a Democracy. Edith Woolsey, Sanford Junior High School, Minneapolis, Minnesota. Consumer Education in the Junior High School. William H. Garrett, Webster Groves High School, Webster Groves, Mo. Mathematics for the College-Preparatory Student, Mr. Browne and Mathematics in a Progressive School, Mr. Taylor.
3. February 22—2:00–4:00 p.m. Non-college-Preparatory Mathematics in the Senior High School. Presiding: Maurice L. Hartung, University of Chicago, Chicago. The Mathematics

Everyone Needs, Kate Bell, Lewis and Clark High School, Spokane, Washington. Mathematics in the Training of Industrial Workers, Clarence Leonard, Southeastern High School, Detroit. Five Years of Experiments in Evanston Township High School, Clara Murphy, Evanston Township High School, Evanston, Illinois. Discussion.

4. February 22—2:00–4:00 p.m. Non-College-Preparatory Mathematics in the Junior High School. Presiding: William Betz, Rochester, New York. The Changing Junior High School and Non-college Mathematics, H. Van Engen, Iowa State Teachers College, Cedar Falls, Iowa. Cultural Material, Ruth Lane, University of Iowa, Iowa City, Iowa. Do We Try to Teach Too Many or Too Difficult Topics, or Both?, Hildegarde Beck, McMichaels Intermediate School, Detroit. Mathematics for the Capable Pupil Not Going to College. Florence Brooks Miller, Shaker Heights, Ohio. Discussion.
5. February 23—10:00–11:45 a.m. What Mathematics Can Contribute to the Education of Modern Youth as Interpreted by Private Schools. Presiding: Dr. M. F. Rosskopf, John Burroughs School, St. Louis Co., Missouri. Mr. Gordon Browne, St. Louis County Day School, St. Louis Co., Missouri. Mr. Lewis Taylor, North Shore Country Day School, Winnetka, Illinois. Miss Ruth Lane, University High School, Iowa City, Iowa. Mathematics in University High Schools.
6. February 23—10:00–11:45 a.m. What Mathematics Can Contribute to the Education of the Investor and the Contractor. Presiding: C. A. Smith, Southwestern High School, St. Louis. Practical Applications of Surveying, Charles A. Hutchinson, University of Colorado, Boulder, Colorado. Education of the Future Investor. From the Viewpoint of the Broker, H. Lloyd Kelley of the firm of Paul Brown and Co., Brokers, St. Louis. From the Viewpoint of the Banker, Byron Moser, President, Mutual Bank and Trust Company, St. Louis. From the Viewpoint of the Insurance Man, Fred T. Rench, President St. Louis Life Underwriters Association and General Agent National Life Insurance Co. of Vt.
7. February 23—3:15–4:30 p.m. Demonstration Lesson. Presiding: Jesse Osborn, St. Louis. Social Mathematics in the High School, Clarabel Parks, Beaumont High School, St. Louis.
8. February 23—3:00–5:00 p.m. Special Conference on Social Mathematics in Grades Eleven and Twelve, An Exchange of Experiences by those Who Have Taught or Designed Such a Course. Presiding: Raleigh Schorling, University of Michigan, Ann Arbor, Michigan. A limited-group meeting. Secure admission by writing or seeing Dr. Schorling or Dr. Christofferson.

IV. Teacher Education Programs

1. February 22—10:15–11:45. The Training of Teachers of Algebra. Presiding: G. H. Jamison, State Teachers College, Kirksville. The Academic Preparation of Teachers of Algebra, J. O. Hassler, Oklahoma University, Norman. The Professional Preparation of Teachers of Teachers of Algebra, A. R. Congdon, University of Nebraska, Lincoln.
2. February 22—2:00–4:00 p.m. The Training of Teachers of Geometry. Presiding: L. H. Whitcraft, Ball State Teachers College, Muncie. What General Knowledge and Skills Should the Teacher Training Program Provide?, L. D. Haertter, John Burroughs School, St. Louis. What Mathematical Knowledge and Abilities Should the Teacher Training Program Provide in Fields Other Than Geometry?, Gertrude Hendrix, Eastern Illinois State Teachers College, Charleston. What Specialized Knowledge Should the Teacher Training Program Provide in the Field of Geometry?, P. D. Edwards, Ball State Teachers College, Muncie. What Professional Knowledge and Experiences Should the Teacher Training Program Provide?, C. N. Mills, Illinois State Normal University, Normal.
3. February 23—10:00–11:45 p.m. The Training of Teachers of High School Arithmetic and General Mathematics. Presiding: F. L. Wren, Peabody College for Teachers, Nashville. Training Teachers of High School Arithmetic, Judson W. Foust, Central State Teachers College, Mt. Pleasant, Michigan. Training Teachers of General Mathematics, Charles H. Butler, Western State Teachers College, Kalamazoo, Michigan. Training Teachers of Mathematics to Comprehend and Appreciate the Interdependence of Different Mathematical Fields, Edwin A. Beito, University of Wichita, Wichita, Kan.

EDITORIAL

THE MAY 1926 number of THE MATHEMATICS TEACHER was issued in honor of Professor David Eugene Smith, who in February of that year retired from active service at Teachers College, Columbia University. At that time his students and colleagues presented a portrait of Professor Smith to the college and gave a dinner in his honor. They endeavored to express to him their appreciation of the uniqueness of his work and their regard for him as a teacher and friend. Professor Smith's influence on the teaching of mathematics had been such that it seemed appropriate then to the editors of THE MATHEMATICS TEACHER to bring together in one number of the magazine the interpretations of his work and contributions as they were made on these occasions.

The frontispiece of the magazine in May 1926 was a photograph of the oil painting of Professor Smith made by Leo Mielziner and presented to Teachers College by his friends on April 27, 1926. The magazine also contained two articles by Professor Smith that had not hitherto been generally accessible, "The Call of Mathematics" and "The Poetry of Mathematics." Here too were published the addresses made at the dinner in Professor Smith's honor by Professor Clifford B. Upton, Toastmaster, Professor Lao G. Simons, Dr. George A. Plimpton, William E. Stark, Miss Charlotte Huber, Mrs. James Pilliod, Professor Cassius J. Keyser, and finally Professor Smith.

The occasion of the presentation of the portrait was also reported in this number. Addresses were delivered by Professor C. T. McFarlane, on behalf of the intramural donors; Dr. George A. Plimpton, on behalf of the extramural donors; Mr. Mielziner, the artist; and Dean Russell, on behalf of the faculty. The addresses on the two occasions as recorded in this number, constitute a documentary summary of Professor Smith's professional life and the appreciation of that life by those whom he influenced most.

Finally the number closed with a condensed list of Professor Smith's publications compiled by Professor Jekuthial Ginsburg. Though impressive this list is only suggestive of the depth and breadth of Professor Smith's scholarship and interests.

It now gives THE MATHEMATICS TEACHER great satisfaction and pleasure to honor Professor Smith once more upon the occasion of his eightieth birthday which will occur on January 21, 1940. So we hereby tender him our congratulations and best wishes for a happy birthday and many happy returns of the day. Nothing that we can say will add to the esteem and love in which Professor Smith is already held by mathematics teachers everywhere. His influence upon the content and teaching of mathematics has been profound.

It may be of interest to our readers to know that in the last three years, Professor Smith has made trips to Russia, to South Africa, to South America, and the last time the editor had occasion to talk to him, he was contemplating another journey to the Vale of Kashmir. Professor Smith is in good health and is still active in working at many things in which he has always been interested.

The frontispiece of this issue is an excellent photograph of Professor Smith and will be of interest to all his friends as well as to those of our readers who have not had the good fortune to meet him personally.

W. D. R.

◆ IN OTHER PERIODICALS ◆

By NATHAN LAZAR
The Bronx High School of Science, New York City

1. Bergen, M. C., "Misplaced Mathematical Recreations." *School Science and Mathematics*, 39: 766. November, 1939.

The writer argues that the tendency to introduce puzzles, trucks, and brain-twisters may not accomplish the results desired for uninterested students. "The interested pupils must be taught to work at full capacity, they must be trained to be leaders. Tricks and puzzles will be interesting to them and they should have them. But the proper place for them is in the mathematics club usually attended by members of their group. . . . The other pupils cannot be aided by puzzles. The subject is puzzle enough. . . . Practice in puzzle-solving makes for proficiency in puzzle-solving, but in nothing else."

2. Corliss, J. J., "If Two External Bisectors Are Equal, Is the Triangle Isosceles?" *School Science and Mathematics*, 39: 732–735. November, 1939.

An extension of Stewart's theorem is used to determine the conditions under which an answer may be found to the proposed question.

3. Escott, Edward B., "Division by Iteration." *The Mathematical Gazette*, 23: 241–245. July, 1939.

The advantages of the method of division by iteration are twofold: (a) substitution of a repetition of a number of easy operations for a more difficult one, and, (b) the fact that at each operation the work starts at the beginning so that errors (except in the last operation) are automatically corrected. The following processes are described in some detail: division by 9, 99, 19, 11, or 71; division by a number ending in 3 or 7; division by 12, 15, 125, 1125, 75, and 875.

4. Georges, J. S., "Mathematical Principles in the Junior High School." *School Science and Mathematics*, 39: 736–749. November, 1939.

The writer envisages the function of mathematics under two categories: unique and general.

The following are listed as the unique educational functions of mathematics: rationalization of the number concept, mastery of the fundamental operations, development of spatial concepts, variables and relationships, intelligent interpretation of tables, working knowledge of graphs, knowledge of symbolic representation, knowledge of the process of symbolic representation, knowledge of the process of measurement, properties of the geometric figure, and the development of efficient reasoning.

The following are enumerated under the heading "General Educational Functions of Mathematics": Intelligent attitude toward problems, toward order, toward laws, toward relationships, toward transformations, toward invariants, toward the principle of representation, toward special cases, and toward generalizations.

5. Goodstein, R. L., "Rational Triangles." *The Mathematical Gazette*, 23: 264–267. July, 1939.

Gauss gave a formula for finding triangles with integral sides and circumradius. The present author proposed a general formula for finding prime solution: that is a formula giving integral sides and circumradius, a, b, c, R, which have highest common factor unity. Another problem dealt with is to find three or more rational right-angled triangles with equal areas.

The usefulness of such formulas for making up exercises suitable for pupils in elementary geometry is obvious.

6. Krathwohl, William C., "Using and Preparing a Mathematics Exhibit." *School Science and Mathematics*, 39: 702–706. November, 1939.

"Model making is a valuable teaching aid in that it satisfies some of the fundamental desires of childhood and assists children to visualize.

"Children usually have much more ability to select models and to make them than their teachers give them credit for.

"Interest can be aroused through the skillful use of competition and the maintenance of prestige.

"The keynotes of a successful exhibit are simplicity, ease of observation, and attractiveness."

7. Lawson, G., "School Algebra of the First Year." *The Mathematical Gazette*, 23: 246–250. July, 1939.

A first lesson on negative numbers. Just as a fraction is treated as a number pair, and the complex number is considered an ordered pair, so should the negative number be considered as a number pair. The advantages of such an approach are pointed out.

8. Pillans, Harry T., "Common Pupil Difficulties with Basic Concepts of Logarithms." *School Science and Mathematics*, November, 1939.

A diagnosis and cure of the following four difficulties which students commonly have with the logarithms of decimal fractions:

(a) The apparent incongruity of negative characteristics and positive mantissas.

(b) The true nature of the operation of the division of such a logarithm by the index of a root.

(c) The meaning of the logarithm of such a logarithm.

(d) The confusion of conceiving a logarithm as an exponent that requires that the base be taken a fractional number of times as a factor.

9. Purdy, C. R., "Points Where the Arithmetic Teacher Can Prevent Algebra Difficulties." *School Science and Mathematics*, 39: 768–769. November, 1939.

Three places are pointed out where the teacher of arithmetic can build for the subsequent work in mathematics without detracting in the least from the subject of arithmetic.

(a) Do not use the signs of process in practice exercises, particularly subtraction.

(b) Avoid using formulas with multiplication and division signs rather than the accepted practice of writing a multiplier adjacent to the multiplicand, and divisors in denominators.

(c) Teach the student to evaluate formulas according to the following procedure: write the formula, write the known facts, substitute in the formula, reduce the resulting expression.

The exact way in which each of the above procedures will benefit the study of algebra is explained in some detail.

10. Weis, Max F., "The Math Club—Streamlines." *High Points*, Vol. 21, No. 8, pp. 74–77. October, 1939.

A description of the activities of a mathematics club which has for its aim "Let's have some fun from math." The following are some of the guiding principles:

(a) The program must appeal to the slow, average, and bright students of mathematics.

(b) There must be a maximum amount of participation by the club members.

(c) The program of every meeting is planned by a steering committee.

(d) Sources of activities should be used that are outside the class room, such as radio, news events, popular games, modern books.

A description is also given of a few programs that have been presented and received with interest and enthusiasm: math information please, math scavenger hunt, logical analysis, mathematics in science, mathematical guggenheim, math bingo, mathematical and non-mathematical fallacies, topolunacies, the story of number dramatized, Professor Quiz—in mathematics, mathematical charades, lives of famous mathematicians dramatized in living newspaper style, and skutsophrenia—short skits depicting math boners.

Save These Dates!

February 22 and 23, 1940—St. Louis, Missouri

Meeting with the American Association of School Administrators and its allied departments and organizations.

Theme: Mathematics for the "Other-than-College-Preparatory" Student.

Headquarters: Hotel Chase. (Get in your reservations at once.)

Program for this really *National* meeting will be found on pages 41 and 42.

June-July, 1940—Milwaukee, Wisconsin

Theme, headquarters and program not yet determined. Suggestions welcomed.

Suggestions and questions should be directed to E. A. Katra, Executive Secretary N.C.T.M., 525 West 125th Street, New York City.

NEW BOOKS

Mathematical Recreations and Essays. By W. W. Rouse Ball. Eleventh edition, revised by H. S. M. Coxeter. The Macmillan Company, 1939. xvi+418 pp. Price, $2.75.

This revised edition is for the most part the tenth edition reprinted, and most teachers are familiar enough with the old edition to make further comment here unnecessary. However, the revised edition has omitted three chapters of the old edition, namely, "String Figures," "Bees and Their Cells," and "Mechanical Recreations." Chapter II has been radically changed as has also Chapter VIII on "Map-Colouring Problems." A great deal of Chapter III on "Geometric Recreations" and Chapter VII on "Magic Squares" is new; Chapter XII has been broken up and distributed among the first, third, fourth, and eleventh chapters. Finally, Chapter V on "Polyhedra" is new, and Chapter XIV on "Cryptographs and Ciphers" has been completely rewritten. Dr. Coxeter has been ably assisted in this revision by Professor D. H. Lehmer, Abraham Sinkov, a cryptanalyst in the United States War Department, by J. M. Andreas, and P. S. Donchian.

This new book will be of great interest and help particularly to secondary teachers of mathematics.

W. D. R.

Fabre and Mathematics and Other Essays. By Lao Genevra Simons. *Scripta Mathematica*, 1939. 101 pp. Price, $1.00.

Often fine creative work is done through the medium of a short form like the essay, the short story, or the magazine article. These, occurring usually in periodicals, are somewhat inaccessible, perishable, or hard to store. Fortunately, collections of them can be made in book form, and that is what has been done with four essays of Professor Simons in this little volume, Number Four, of The Scripta Mathematica Library.

Three of these historical and biographical articles have already appeared in *Osiris*, *Isis*, *Scripta Mathematica*.

The first, "Fabre and Mathematics" is a delightful account of the discovery by the naturalist Jean Henri Fabre of the fascination and power of mathematics. Liberal quotations from Fabre's own account of his experiences make vivid the thrill that possessed him.

The activities, researches, and correspondence with great mathematicians of von Humboldt are recountered in the second article "The Interest of Alexander von Humboldt in Mathematics."

"The Influence of French Mathematicians at the End of the Eighteenth Century Upon the Teaching of Mathematics in American Colonies" deals with "the great trio," Lagrange, Laplace, and Legendre. Prejudices against all things British turned American scholars toward the French influence in the early history of this country. The story of the contribution of the three French giants to the development of mathematical scholarship in America is interestingly told.

The results and conclusions of Professor Simons' researches in the records of the teaching of mathematics in the early history of this country are in the final article "Short Stories in Colonial Geometry." Here we learn that although algebra and geometry did not amount to much in early American colleges, they did amount to more than many scholars have thought or known.

W. D. R.

Learning Activities in Elementary Algebra. By Earl J. Burnett and Regina Grosswege. Edited by Halbert C. Christofferson. College Entrance Book Company, Inc., 1939. 228 pp. Price, $.88.

Learning Activities in Plane Geometry. By Earl J. Burnett and William E. Batzler. Edited by Halbert C. Christofferson. College Entrance Book Company, Inc., 1939. 248 pp. Price, $.90.

These two workbooks have been produced under the editorship of Professor H. C. Christofferson, of Miami University, Oxford, Ohio, president of the National Council of Teachers of Mathematics. It is not the business of a reviewer to turn detective and try to discover and measure the contribution of each author or the editor working on the series. It is enough to say that the work as a whole is a credit to everyone involved and of a quality consistent with the reputation of the well-known national authority on mathematics teaching, Professor Christofferson.

Both books are complete enough to be used without a textbook and conventional enough to be used with any textbook. However, they contain frequent departures from the commonplace in clever presentations of ideas. Each unit con-

tains two introductory pages, one with an attractive halftone illustration and a discussion of modern, practical applications of the topic to come; the second, giving the historical origins and development of the subject.

Three grades of problems take care of individual differences. Every unit contains topical and cumulative reviews. A Manual of Mastery Tests accompanies each book. The books include a scoring chart and a page for recording scores. The pages are perforated and punched so they may be torn from the book, passed to the teacher for corrections, and placed in a loose leaf binder.

The content of the algebra lies for most part within the usual college-preparatory, first year high school course. Arithmetic drills, social mathematics, and such are pretty much ignored. Some of the more difficult exercises, while remaining within the first year algebra range of difficulty, anticipate forms and notations of later courses.

The geometry workbook is in line with those progressive trends of the past few years which its authors and editor have had a share in shaping.

E. W.

Mathematical Snapshots. By H. Steinhaus. G. E. Stechert and Co. 135 pp. Price, $2.50.

This little book is equipped with red-and-green spectacles for looking at its anaglyphs, a dodecahedron, and a set of cards which handled skillfully give the operator four different motion pictures. It is a humorous, casual presentation of a number of mathematical facts, curiosities, and classical problems, with illustrations on every page and a light running explanation.

It does not pretend to be a systematic text on any part of mathematics. Mathematical recreation is the main object of this fine book. It would be a good book to have in the mathematics classroom or for the use of a mathematics club.

Space does not permit us to give an elaborate description of the details, but one will find many interesting bits of material that will be of interest to pupils such as soap-bubbles, crystals, möbius bands, polyhedra and the like.

W. D. R.

Graphic Presentation. By Willard Cope Brinton. Brinton Associates, 1939. 512 pp. Price, $5.00.

Mathematics teachers will not be able to claim a monopoly of benefits from this striking book on graphs, whether making or interpreting them. Glancing through this book, one becomes convinced that many more facts and issues can be presented graphically than one had imagined. The social studies teachers, to mention one group, will find a new program of activities for enriching their work.

Here are graphs in impressive variety representing quantities and relationships in many fields—trade, agriculture, mining, manufacturing, management. Here one sees the practicality of mathematics and its close association with all parts of living and doing.

Imaginative teachers have always allowed themselves to proceed beyond the requirements of syllabi and the suggestions of textbooks in their work on graphs. In general, though, emphasis has been laid on the mathematical exactness of students' work. This is as it should be in mathematics.

However, there has been some meagreness in variety of materials presented and methods of dealing with them, and the graphs of advertisers have run far ahead of the textbooks in appeal. The graphs in *Graphic Presentation* tell their story simply and with dash and verve. The textual comment gives the inside story, as it were, of each graph and discloses the shrewd secrets that you may not think of when you see the disarming simplicity of the graphs.

With the growing use of graphs in popular reading matter of all kinds, the average citizen must be equipped not only to understand but to criticise and weigh the significance of the messages of graphs. Understanding graphs is like understanding music or art. After you have made them yourself, you are more competent to judge the work of others. For that reason if for no other, it will be worth-while to give your students as wide an experience as possible with graphs.

Color has been used brilliantly throughout this book. Surely that will cause the addition of a box of colored pencils to the common equipment of many mathematics classrooms.

E. W.

Memo for the Second Semester:

The 80% can't use IVORY TOWER *Mathematics*.

"The 80%" is a symbol. It stands for the great majority of pupils, who will be short-changed educationally by formal or "drill" mathematics. They will be short-changed equally by mere "simplified" mathematics. They will never be enthusiastic about either type of work. They need mathematics that is presented in terms of their present interests, and that deals with their future personal and social problems as citizens. A course that backs up the work these pupils do in social-studies, consumer, and home-economics courses not only will teach them mathematics, but at the same time will do its part to make them better citizens. The five Boyce-Beatty Units encourage pupils to develop their powers of quantitative thinking in major areas of consumer problems and citizenship. On the 7th to 9th grade level, they are for both "the 80%" and "the 20%." On the senior-high level they are exceptionally effective for retarded classes.

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The mathematics work in the **HEALTH UNIT—The Economics of Health,** is developed largely around the central theme of buying health. The bulk of this Unit deals with the buying of medical and health services, private and public; the relation to health of the food, clothing, and housing that we buy; our extravagant buying for health as a result of misleading advertising; etc. (List price of this Unit, **84¢**)

LEISURE UNIT

In the **LEISURE UNIT—The Economics of Leisure Activities,** the mathematics exercises are centered around the purchasing of the myriad accessories of our leisure pursuits—and, in fact, the purchasing and financing of the leisure activities themselves. That includes instalment buying of cars, radios, etc.; planning and buying food for parties; how to handle money and keep accounts for group affairs, and procedures in purchasing and making payment, etc. List price of this Unit, **84¢**)

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